Article

The impact of risk and mobility in dualistic models: Migration under random shocks

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A B S T R A C T

In this paper we present and confront the expected outcome of an increase in risk on the regional or sectoral allocation of labor force and employment. The basic frameworks are the benchmark dualistic scenarios. A single-input analysis of a homogeneous product economy is provided. Uncertainty is modeled as localized Bernoulli random experiments, additively affecting either labor demand or labor productivity, unilaterally, or in a perfectly (positive and negative) correlated fashion in both regions providing a stage from which conclusions on the expected consequences of random shocks (or of changes in workers’ heterogeneity) to the economy can be drawn. A (deterministic) differentiated natural appeal of — an intrinsic imbalance between, a compensating income differential required by affiliates of one sector—the two regions is allowed to interact with equilibrium formation.

We report the main effects on equilibrium local expected wages, supply, employment and aggregate welfare surplus of a unilateral as well as a simultaneous increase of labor demand dispersion in the (a) basic two-sector model in four different scenarios: free market; partial (one-sector) coverage with perfect inter-sector mobility; partial (one-sector) coverage with imperfect mobility (Harris-Todaro); multiple (two-sector) coverage with imperfect mobility (Bhagwati-Hamada).

Importance of convexity of local labor demands was invariably recognized. A localized increase in risk does not always repel the labor force in the long-run. This statement would hold even if individuals were not risk-neutral, as assumed in the research.

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El impacto del riesgo y la movilidad en los modelos dualísticos: migración bajo shocks aleatorios

R E S U M E N

En este documento presentamos y confrontamos los resultados esperados del incremento del riesgo de la distribución sectorial de la mano de obra y el empleo. Los marcos básicos son los escenarios dualísticos de referencia. Se aporta el análisis con un solo input de una economía de producto homogénea. La incertidumbre se modela como experimento localizado y aleatorio de Bernoulli, afectando en forma acumulada a la demanda de mano de obra o a la productividad laboral de modo unilateral, o de manera perfectamente correlacionada (positiva o negativamente) en ambas regiones, proporcionando un escenario desde el que pueden extraerse conclusiones sobre las consecuencias que se esperan de los shocks aleatorios (o de los cambios de heterogeneidad de los trabajadores) para la economía. Se permite la interacción, entre las dos regiones, del llamamiento natural diferenciado (determinista), el desequilibrio intrínseco y la compensación del diferencial de los ingresos requerida por los afiliados de un sector, con formación de equilibrio.

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1. Introduction

The aim of this research is to contrast the expected long-run impact of exogenous uncertainty on labor force flows and expected wages under alternative scenarios of institutional wage setting and barriers to mobility. The study of the matter would be a-temporally relevant; to the extent that migration issues are being discussed among the EU member states, and given the recent enlargement to new economies with different market rules on the one hand, and security profiles on the other, the layout of theoretical foundations for the understanding of those effects would appear as a timely exercise.

The basic structures chosen to replicate the effects of uncertainty were simple dualistic models in the tradition of Harris-Todaro (1970) rural-urban migration analysis. A good survey of theoretical literature can be found in Bhattacharya (1993). The principles behind its workings became widespread in the study of labor market regional as also sector —occupation, profession— allocation and under minimum or other wage legislation or restrictions. Examples of these can be found in Mincer (1976); McDonald and Solow (1985), Fields (1989), Brown, Gilroy and Kohen (1982). A survey of segmented labor markets can be seen in McNabb and Ryan (1990); and the applications of the theory with microfoundations for several dualistic structures can be found in Saint-Paul (1996).

We follow the cases contrasted in Martins (1996), inspecting the consequences of introducing a local stochastic noise of various nature in each of the scenarios, these differing according to the degree of mobility across the two sectors —of whether there is immediate access to the other region jobs or not—, and to whether any or both regions or sectors are subject to an (also exogenous) institutional wage floor.

Total —national, worldwide according to context we may wish to simulate— labor force supply is assumed perfectly inelastic. Workers choose location, or sector affiliation, maximizing the expected wage —risk-neutrality allows us to concentrate on how technology characteristics rather than risk-aversion of the individuals (the role of individual’s risk-aversion on migration decisions has been studied before and was surveyed by Stark (1991). It was our purpose to generate, thus, other type of conclusions) of the population affect the market equilibrium responses—.

Exogenous uncertainty itself may interplay with the underlying local technologies in different ways. Two environments were always simulated: when uncertainty works as an (null expected mean) added noise to local labor demand —quantity uncertainty; and added to the inverse labor demand —that is, to labor productivity. For simplicity, such noise was modeled as a binary random shock —conclusions shouldn’t change qualitatively if we assumed a general distribution— and we inspected the effect of an increase in its variance maintaining the mean constant. (That is a general conclusion in the inspection of the effects of uncertainty on the risk-premium (Martins, 2004)).

As the prototype economy has two regions or sectors, perfectly (positively and negatively) correlated increases in local risks were also simulated. Changes in uncertainty can also mimic changes in the degree of heterogeneity of the labor force —or local productive ability to cope with them—.

Being mobility of major concern in the analysis, two extreme cases of “barriers to adjustment” were also thought to be important in the inquiry: either adjustment to uncertainty is assumed to be immediate to the random shock. Then, the long-run equilibrium differs according to which, and is formed after the, exogenous impulse is observed —ex-post flexibility—. Or binding location/sector affiliation decisions are taken before the actual risk realization —ex-ante location choices—. (See Aiginger (1987) and Martins (2004a) for a survey and appraisal of the effects of uncertainty on production outcomes under the two contexts). Obviously, the latter stages wage dispersion more appropriately if the local wage is left as market determined rather than institutionally fixed.

After notation is briefly settled in Section II, we depart from the benchmark case —free market with perfect mobility across regions or sectors—, outlined in Section III. In Section IV, partial coverage with perfect mobility —i.e., people not employed in the primary sector can immediately get a job in the secondary sector and wait there for an opportunity to switch, and thus, there is (again) no unemployment generation— is introduced. In Section V, a version of the Harris-Todaro model —with imperfect mobility and institutionally fixed wage in one of the sectors— is inspected. In Section VII, the Bhagwati-Hamada economy —with two covered regions or sectors— is forwarded. The exposition ends with a concluding appraisal in Section VII.

2. Notation

There are two regions —or two sectors— and a fixed exogenous labor supply, \( L_i^- \). This total labor supply decides whether to locate in region (or affiliate to sector) \( 1 \) or \( 2 \). Denote by \( L_i^- \), local/industry supply in region/sector \( i \). Then:

\[
L_i^- + L_i^+ = L_i^-
\]

In region \( i \), the baseline deterministic aggregate demand function is given by:

\[
L_i = L_i(W_i), \quad i = 1, 2
\]

A non-positive slope —that is, \( \frac{dL_i}{dW_i} \leq 0 \) — is always assumed. Denote the corresponding inverse demand function by:

\[
W_i = W_i(L_i), \quad i = 1, 2
\]

There are no cross effects, i.e., \( \frac{dL_j}{dW_i} = 0 \) for \( i \neq j \). The wage elasticity of demand of region \( i \) at a particular point of labor demand will be denoted by

\[
s^i = \left( \frac{W_i(L_i)}{L_i} \right) \left( \frac{W_i(L_i)}{L_i} \right) = \frac{W_i(L_i)}{W_i(L_i)} = \frac{L_i}{L_i}. \]

Reportamos los principales efectos sobre los salarios locales previstos en equilibrio, el suministro, el empleo y el excedente del bienestar acumulado de un incremento unilateral y simultáneo de la dispersión de la demanda de mano de obra en el (a) modelo básico bi-sectorial en cuatro escenarios diferentes: mercado libre; cobertura parcial (unisectorial) con movilidad perfecta intersectorial; cobertura parcial (unisectorial) con movilidad imperfecta (Harris-Todaro); cobertura múltiple (bi-sectorial) con movilidad imperfecta (Bhagwati-Hamada). Invariablemente, se reconoció la importancia de la convexidad de las demandas locales de mano de obra. El incremento localizado del riesgo no siempre ahuyenta a la mano de obra a largo plazo. Esta aseveración se mantendría incluso cuando los individuos no fueran neutros al riesgo, según los supuestos de la investigación.

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Let \( Z_i \) be a Bernoulli lottery of null expected value: with probability \( q_i \), it takes value \( s_i \); with probability \( 1 - q_i \), it takes the value \(- q_i \). 
\[
\text{Var}(Z_i) = \frac{q_i s_i^2}{1 - q_i} 
\]
\[
(5)
\]

\( \text{Var}(Z_i) \) increases with \( s_i \) if \( s_i \) is positive, decreases if it is negative. A change in \( s_i \) is modeled as a change in \( Z_i \), increasing with it if \( s_i > 0 \), decreasing if \( s_i < 0 \); we simulate a “mean preserving spread” in the sense of Rothschild and Stiglitz (1970).

Two general risk environments are simulated: an absolute uncertainty \( Z_i \) is either added to local labor demand —and denoted by \( X_i \) — or quantity uncertainty:

\[
L_i = L_i^1(W_i) + X_i \quad \text{and} \quad W_i = W_i^1(L_i - X_i) \quad i = 1, 2
\]
\[
(6)
\]

Or to labor productivity — \( Y_i \):

\[
L_i = L_i^1(W_i - Y_i) \quad \text{and} \quad W_i = W_i^1(L_i + Y_i) \quad i = 1, 2
\]
\[
(7)
\]

Then meaningful effects are found — and, hence, inspected — for environments of:

1) Ex-post flexibility: the individuals make location choices after the observation of \( Z_i \). We will denote the equilibrium wage generated or labor force located in region \( i \) by:

\[
\begin{align*}
X_i &\colon W_i^a \text{ or } L_i^1 - \delta_i^a \text{ when } s_i \text{ occurs;} \\
Y_i &\colon W_i^b \text{ or } L_i^1 - \delta_i^b \text{ when } -\frac{\delta_i^b}{1 - \delta_i^a} \text{ does.}
\end{align*}
\]
\[
(8)
\]

2) Ex-ante location arrangements. The population fixes its own or affiliates to a sector. Once there, if in a covered sector, it suffers random wage fluctuations; if in a covered sector, it is subject to employment availability uncertainty.

The background technology and preferences in the economy are anything but complex: an homogeneous good is produced and consumed in both regions. Identical workers as “land-owners” consume directly what they produce or receive as income: there is no (reason to), nor (need for) money. We allow for regional (sectoral) imbalances in terms of intrinsic resources: location in region 2 systematically provides a income–value differential in a favor of each of its residents (a is allowed to be negative, though).

We will assume further through Sections II to V a subset of the following:

1. individuals are risk neutral and maximize expected income.
2. a. only region \( i \) is affected by risk — \( Z_i = 0 \).
3. b. both regions are affected by perfectly positively correlated uncertainty — \( Z_1 = Z_2 = Z \).
4. c. both regions are affected by perfectly negatively correlated uncertainty — \( Z_1 = -Z_2 = -Z \).
5. a. there is perfect mobility across regions, alternatively.
6. b. job rotation is only accomplished locally or within the industry.
7. a. wage in region \( 1 \) is determined by market conditions, alternatively.
8. b. wage in region \( 1 \) is institutionally determined.
9. a. wage in region \( 2 \) is market determined, alternatively.
10. b. wage in region \( 2 \) is institutionally determined.

Given assumptions 1 — always assumed — and the fact that a differential favors (for \( a > 0 \) region \( 2 \), an equilibrium will be such that \(^1\):

\[
E[W_i^a] = a + E[W_i^b] \quad j = s, A, B
\]
\[
(9)
\]

where * denotes an ex-ante decision context. Worker flows will exist till equalization of expected wage net of the compensation amount in the two regions.

Assumptions 2 stage different types of local risks. 2.a is useful to analyze the impact of unilateral risk level —inducing conclusions expected to be generalizable to situations where both regions are subject to uncertainty and the change in risk occurs in only one of them. 2.b and 2.c allow for a relation between uncertainty movements in the two regions— under ex-ante location commitment, the sign of the pertaining correlation becomes in some sense redundant to the determination of labor force flows.

Assumptions 3 to 5 characterize the mobility environment. Different combinations of alternatives a and b generate backgrounds of benchmark dualistic models that we shall stage. For instance, 3.a. insures that expected unemployment in the economy will be zero —provided that the wage is market determined in at least one of the regions—.

The fixed institutional wage mimicks a minimum wage in the region.

3. Competitive labor markets under perfect mobility

Assume 3.a. 4.a and 5.a of the previous section. Then, under certainty equilibrium, labor flows will be expected between the two regions while equalization of individuals’ welfare is not achieved, i.e., till:

\[
W_1 = a + W_2
\]
\[
(10)
\]

\[
L^1(W_2 + a) + L^2(W_2) = L^2(L^1 - L_2) + a
\]
\[
(11)
\]

Differentiating equation (11), a change in the relative differential favoring region 2 will imply changes in wages and employment allocation according to:

\[
\partial W_2 / \partial a = -L^1(W_2 + a) / [L^1(W_2 + a) + L^2(W_2)] < 0
\]
\[
(12)
\]

\[
\partial W_1 / \partial a = L^2(W_2 + a) / [L^1(W_2 + a) + L^2(W_2)] > 0
\]
\[
(13)
\]

\[
\partial L_1 / \partial a = 1 / [W_1^1 L_1' + W^2(L^1 - L_2)] < 0
\]
\[
(14)
\]

where the inequalities follow from the assumed negative slope of the labor demand curves (and thus negative slopes of the inverse labor demand).

The wage bill —and the expected wage in the economy— will react according to:

\[
\partial(W_1 L_1 + W_2 L_2) / \partial a = L_1 \partial W_2 / \partial a + L_2 + a \partial L_1 / \partial a
\]
\[
(15)
\]

It will increase (decrease) with \( a \) if

\[
-1[L^1(W_2 + a) / L^1(W_2 + a) - L^2(W_2) / L^2(W_2)] < \langle -a \rangle
\]

or

\[
[L^1(W_2 + a) / L^1(W_2 + a) - L^2(W_2) / L^2(W_2)] [L^2(W_2) / L^2(W_2)] > \langle -a \rangle
\]

i.e., the differentiation between the absolute value of the semi-elasticity of labor demand in region 2 and that of 1 over their product —or the difference between the inverse of the semi-elasticities of those labor demands— exceeds (is smaller than) \( a \), the per capita external externality favoring inhabitants of region (or professionals of sector) 2.

Yet, the individual’s worker well-being is in fact measured by the equilibrium value of \( W_1 \) — and aggregate workers’ welfare value \( L^1 - W_1 \), rising with \( a \).

Proposition 1.1.1. Under free market, the usual dualistic model will result in equalization of wages net of compensating differentials across regions or sectors and there will be no unemployment.

1.2. An increase in local externalities favouring one of the regions will attract the labor force to the latter, depress the local wage and cause unemployment in the former.
wage and increase it in the other region. The average wage in the economy will increase (decrease) if the difference between the absolute value of the semi-elasticity of labor demand in the favored region and that of the other over their product exceeds (is smaller than) the per capita externality differential. Aggregate welfare will increase.

3.1. Quantity Uncertainty

3.1.1. Ex-post flexibility

Consider case 2.a: only region i is affected by uncertainty. The labor force is:

\[ W_i^t = a + W_i^s, \quad j = A, B \]

(16)

Either \( s_i \) is observed - which occurs with probability \( q_i \) - and

\[ L^1(W_2^a + a) + L^2(W_2^B + a) + s_i = L_i^- \]

Then, differentiating:

\[ \partial W_2^a / \partial s_i = -1 / [L^1(W_2^a + a)'] + L^2(W_2^B) \] (17)

or, with probability \((1 - q_i)\), \(- q_i / (1 - q_i)\) is and:

\[ L^1(W_2^a + a) + q_i s_i / (1 - q_i) + L^2(W_2^B) = L_i^- \]

And differentiating:

\[ \partial W_2^a / \partial s_i = q_i / [L^1(W_2^a + a)'] + L^2(W_2^B) \]

As \( \partial W_2^a / \partial s_i > 0 \) (and/or \( W_2^B / \partial s_i < 0 \)), for \( s_i > (<)0, W_2^a > (<)W_2^{A,B} \). On average:

The expected wage responds to uncertainty according to:

\[ \partial E[W_2^i] / \partial s_i = q_i \partial W_2^a / \partial s_i + (1 - q_i) \partial W_2^B / \partial s_i \]

\[ \partial E[W_2^i] / \partial s_i = q_i (L^1(W_2^a + a) + L^2(W_2^B)) \]

(18)

For \( s_i > (<)0, W_2^a > (<)W_2^B \) and \( \partial E[W_2^i] / \partial s_i > (<)0 \) if \( L^1(W_2^a + a) + L^2(W_2^B) \) rises with \( W_2 \). That occurs when (around \( a = 0 \), the sum of both labor demands are (is) convex in \( W_2 \) - that is, \( L^1(W_2^a + a) + L^2(W_2^B) \) increases if \( W_2 \) increases). \( E(W_2^i) \) decreases with uncertainty (rises with \( s_i \) if positive, decreases if negative) when both (around \( a = 0 \), the sum of labor demands are (is) concave.

Overall, no unemployment is ever generated in the economy. Labor force allocation changes both immediately after a shock, as in expected terms when \( s_i \) rises. When uncertainty rises in region 1:

\[ \partial E[L^1(W_2 + a) + X_1] / \partial s_i = -\partial E[L^1(W_2^B) + X_1] / \partial s_i \]

\[ \partial E[L^1(W_2 + a) + X_1] / \partial s_i = -q_i \partial L^1(W_2^a) / \partial s_i + (1 - q_i) \partial L^1(W_2^B) / \partial s_i \]

\[ \partial E[L^1(W_2 + a) + X_1] / \partial s_i = -q_i (L^1(W_2^a) + L^2(W_2^B)) \]

(19)

For \( s_i > (<)0, W_2^a > (<)W_2^B \) and \( \partial E[L_1] / \partial s_i > (<)0 \) if \( L^1(W_2 + a) / L^2(W_2) \) decreases with \( W_2 \).

If it rises in region 2:

\[ \partial E[L^1(W_2 + a)] / \partial s_2 = -\partial E[L^2(W_2 + X_2)] / \partial s_2 \]

\[ = q_2 \partial L^1(W_2^a + a) / \partial s_2 + (1 - q_2) \partial L^1(W_2^B + a) / \partial s_2 \]

\[ = q_2 L^1(W_2^a + a) \partial W_2^a / \partial s_2 + (1 - q_2) L^1(W_2^B + a) \partial W_2^B / \partial s_2 \]

\[ = q_2 [(L^1(W_2^a) / L^1(W_2^a + a)'] - L^2(W_2^B) / L^1(W_2^a + a)] ] \]

(20)

\[ L^1(W_2^a + a) \partial W_2^a / \partial s_2 + (1 - q_2) L^1(W_2^B + a) \partial W_2^B / \partial s_2 \]

\[ L^1(W_2^a + a) \partial W_2^a / \partial s_2 + (1 - q_2) L^1(W_2^B + a) \partial W_2^B / \partial s_2 \]

For \( s_2 > (<)0, W_2^a > (<)W_2^B \) and \( \partial E[L_2] / \partial s_2 > (<)0 \) if \( L^1(W_2^a + a) / L^2(W_2^B) \) decreases with \( W_2 \). Around \( a = 0 \), that requires that \( L^1(W_2^a + a) / L^2(W_2^B) \) decreases with \( W_2 \). Then:

\[ E(L_2) \] increases (decreases) with uncertainty (rises with \( a < 0, W_2^a < (<)W_2^B \) decreases (increases) with uncertainty (rises with \( s_2 < 0, W_2^a < (<)W_2^B \)).

With negatively correlated uncertainty - hypothesis 2.c.-, the wage is invariant to and stabilized at the level of:

\[ L^1(W_2 + a) + L^2(W_2) = L_i^- \]

(21)

Employment in each region fluctuates mimicking the local shock. Expected local employment does not vary with uncertainty.

Summarizing:

**Proposition 2:** Under free market, quantity uncertainty and ex-post location flexibility:
2.1. A unilateral or generalized and positively correlated across regions increase in risk will result in a rise (decrease) in the expected "net" wage if the sum of labor demands is convex (concave).

2.2. A unilateral or generalized and perfectly and perfectly correlated across regions increase in risk will result in a rise (decrease) in the expected employment in a region if the ratio of its labor demand slope to the other region's decreases (increases) with the wage rate, that is, if the symmetric of the Arrow-Pratt measure of risk aversion of the local demand (negatively sloped) is smaller larger) than that of the other regions.

2.3. Perfectly and negatively correlated labor demand shocks will result in equal employment movements after a shock, neutralizing any effect on the expected wages or local labor force in the economy.

3.1.2. Ex-ante location arrangements

Consider case 2.a: only sector i is affected by uncertainty. The labor force settles or affiliates before observing \( X_i \); after, when it makes location changes to region j, it does not know which \( X_i \) it is going for. For equilibrium:

\[
q_i W_i'(L_i - s_j) + (1 - q_i) W_i'(L_i + \frac{qs}{1-q}) = W_i'(L_i - L_i) \pm a
\]

Then, differentiating:

\[
\partial s_i / \partial s_i = q_i [W_i'(L_i - s_j) - W_i'(L_i + \frac{qs}{1-q})] / (q_i W_i'(L_i - s_j) + (1 - q_i) W_i'(L_i + \frac{qs}{1-q}) + W_i'(L_i - L_i))
\]

and

\[
\partial E[W_i] / \partial s_i = \partial E[W_i] / \partial s_i = -W_i'(L_i - L_i) \partial s_i / \partial s_i
\]

The expected wage in the economy moves in the same direction as employment of the region where uncertainty is changing. Employment will flow to the region when local uncertainty rises (decreases) if \( \partial L_i / \partial s_i > (\leq) 0 \) when \( s_i > 0, \partial L_i / \partial s_i < (\leq) 0 \) when \( s_i < 0 \) if \( W_i'(L) > (\leq) 0 \) which occurs, once labor demand is negatively sloped, when \( W_i'(L) > (\leq) 0 \), i.e., when labor demand in region i is convex (concave).

Consider case 2.b: both sectors are affected by simultaneous idiosyncratic shocks. For equilibrium:

\[
q_i W_i'(L_i - s_j) + (1 - q_i) W_i'(L_i + \frac{qs}{1-q}) = q W_i^2(L_i - L_i + s) + (1 - q) W_i^2(L_i - L_i - s) + a
\]

Then:

\[
\partial s_i / \partial s = q_i [W_i'(L_i - s_j) - W_i'(L_i + \frac{qs}{1-q})] / (q_i W_i'(L_i - s_j) + (1 - q_i) W_i'(L_i + \frac{qs}{1-q}) + W_i'(L_i - L_i))
\]

Employment will flow to region 1 when general uncertainty rises (decreases) if \( \partial L_i / \partial s > (\leq) 0 \) when \( s > 0, \partial L_i / \partial s < (\leq) 0 \) when \( s < 0 \) if \( W_i'(L) > (\leq) 0 \) increases (decreases) with q, which occurs, once labor demand is negatively sloped, when \( W_i'(L) > (\leq) 0 \) i.e., when inverse labor demand in region 1 is more convex (concave) than that of region 2. As for the wage:

\[
\partial E[W_i] / \partial s = \partial E[W_i] / \partial s = [q W_i W_i'(L_i - s_j) + (1 - q) W_i W_i'(L_i + \frac{qs}{1-q})] / (q_i W_i'(L_i - s_j) + (1 - q_i) W_i'(L_i + \frac{qs}{1-q}) + W_i'(L_i - L_i))
\]

As \( W_i'(L_i - s_j) > (\leq) 0 \) if \( W_i'(L_i - L_i) > (\leq) 0 \) and has the sign of \( \partial E[W_i] / \partial s \), we expect that the expected wage will rise (decrease) with uncertainty if both inverse demands are convex (concave).

Finally, take case 2.c: both regions are affected by simultaneous symmetrical shocks. For equilibrium:

\[
q W_i^2(L_i - L_i + s) + (1 - q) W_i^2(L_i - L_i - s)
\]

Then:

\[
\partial s_i / \partial s = q_i [W_i'(L_i - s_j) - W_i'(L_i + \frac{qs}{1-q})] / (q_i W_i'(L_i - s_j) + (1 - q_i) W_i'(L_i + \frac{qs}{1-q}) + W_i'(L_i - L_i))
\]

and

\[
[q W_i^2(L_i - L_i + s) + (1 - q) W_i^2(L_i - L_i - s) + q W_i^2(L_i - L_i)]
\]

Approximately —through Taylor expansion to the first-order of the terms in the numerator, \( \partial L_i / \partial s > 0 \) if \( s > 0 \) (and \( \partial L_i / \partial s < 0 \) when \( s < 0 \)) if \( W_i'(L_i)^2 > W_i'(L_i - L_i)^2 \): employment rises in the region which has a more convex inverse labor demand.

As for the expected wage:

\[
\partial E[W_i] / \partial s = E[W_i] / \partial s = [q W_i W_i'(L_i - s_j) + (1 - q) W_i W_i'(L_i + \frac{qs}{1-q})] / (q_i W_i'(L_i - s_j) + (1 - q_i) W_i'(L_i + \frac{qs}{1-q}) + W_i'(L_i - L_i))
\]

Employment will flow to region 1 when general uncertainty rises (decreases) if \( \partial L_i / \partial s > (\leq) 0 \) when \( s > 0, \partial L_i / \partial s < (\leq) 0 \) when \( s < 0 \) if \( W_i'(L) > (\leq) 0 \) increases (decreases) with g, which occurs, once labor demand is negatively sloped, when \( W_i'(L) > (\leq) 0 \) i.e., when inverse labor demand in region 1 is more convex (concave) than that of region 2.
The expected wage increases (decreases) when general uncertainty rises – $\partial E[W_1]/\partial s > (\prec) 0$ when $s > 0$, $\partial E[W_1]/\partial s < (\prec) 0$ when $s < 0$ – if $W_1^1(L + g)/W_2^1(L^1 − L − g)$ decreases (increases) with $g$ or, equivalently, if $W_1^1(L + g)/W_2^2(L^2 − L + g)$ decreases (increases) with $g$. That occurs, around $g = 0$, when $W_1^1(L^1)/W_1^2(L^2)$ is of negative sum of the symmetric of the Arrow-Pratt measures of risk aversion of the inverse demands (that symmetric measuring their concavity) is smaller (larger) than $0$.

**Remark 2:** If the product of the marginal values of the two functions of the same argument, $W_1^1(L^1)W_2^1(L^2)$, increases with it, the sum of the corresponding absolute risk aversion measures:

- is negative and $− W_1^1(L^1) / W_2^1(L^2) < W_2^2(L^2) / W_2^1(L^1) < 0$ when $W_2^1(L^1) > 0$.
- is positive and $W_1^1(L^1) / W_2^1(L^2) + W_2^2(L^2) / W_2^1(L^1) > 0$ when $W_2^1(L^1) < 0$.

**Proposition 3:** Under free market, subject to quantity uncertainty and to ex-ante location decisions:

1. A unilateral increase in risk will result in a rise (decrease) in the expected wages and employment in the affected region if its labor demand is convex (concave).

2. A generalized and perfectly correlated across regions increase in risk will result in a rise (decrease) in the expected employment in the region of more convex (concave) inverse labor demand.

3. A generalized and positively and perfectly correlated across regions increase in risk will result in a rise (decrease) in the expected wages if both labor demands are convex (concave).

4. Perfectly and negatively correlated labor demand shocks will result in a rise (decrease) in the expected wages if the sum of the symmetric of the Arrow-Pratt measures of risk aversion of the inverse labor demands is negative (positive).

### 3.2 Productivity uncertainty

#### 3.2.1 Ex-post flexibility

Consider case 2.a: only region i, is affected by uncertainty, implying a noise around the observed wage distribution – subtracted from the local expected or average wage.

Let $W_i$ be the region affected by uncertainty. Either $s_i$ is observed – which occurs with probability $q_i$ – and

$$L^1(W_2^1 + a − s_i) + L_2(W_2^2) = L^0$$

Then:

$$\partial W_2^1 / \partial s_1 = L^1(W_2^1 + a + s_i)/L^1(W_2^2 + a + s_i)^2 + L^2(W_2^1)^2 > 0$$

Or, with probability $(1 − q_1)$, $− q_1 s_1 / 1 − q_1$ is and:

$$L^1(W_2^1 + a + q_1 s_1) / 1 + q_1 + L^2(W_2^2)^2 = L^0$$

and

$$\partial W_2^1 / \partial s_1 = L^1(W_2^1 + a + q_1 s_1) − 1 / 1 + q_1$$

$$L^1(W_2^1 + a + q_1 s_1^2 + L^2(W_2^2)^2) < 0$$

As $\partial W_2^1 / \partial s_1 > 0$ (and/or $\partial W_2^1 / \partial s_1 < 0$). for $s_i > (\prec) 0$, $W_2^1 > (\prec) W_2^0$. On average:

$$\partial E[W_2]/\partial s_1 = q_i W_2^1 / \partial s_1 + (1 − q_i) \partial W_2^0 / \partial s_1$$

$$= q_i l^1(W_2^1 + a − s_i)/[L^1(W_2^1 + a − s_i) + L^2(W_2^1)]$$

$$− l^1(W_2^1 + a + q_1 s_1) / [L^1(W_2^1 + a + q_1 s_1) + L^2(W_2^1)]$$

(28)

This increases (decreases) with $s_1 > (\prec) 0$, $W_2^1 > (\prec) W_2^0$.

Overall, no unemployment is ever generated in the economy. When uncertainty rises in region 1:

$$\partial E[L^1(W_2^1 + a − s_1)/L^2] = − E[L_2^2(W_2^1)] / \partial s_1$$

$$= − [q_1 l^2(W_2^1) / \partial s_1 + (1 − q_1) l^2(W_2^1) / \partial s_1]$$

(29)

If it rises in region 2, we will have analogous conclusions. Let us define the problem in terms of inverse demands: either $s_2$ is observed – which occurs with probability $q_2$ – and

$$W_2^1(L^1 − L_2^2) = W_2^0(L_2^2) + s_2 + a$$

Then:

$$\partial L_2^2 / \partial s_2 = − 1 /[W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2)] > 0$$

Or, with probability $(1 − q_2)$, $− q_2 s_2 / 1 − q_2$ is and:

$$W_2^1(L^1 − L_2^2) = W_2^0(L_2^2) − q_2 s_2 / 1 − q_2$$

and

$$\partial L_2^2 / \partial s_2 = q_2 s_2 / [W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2)] < 0$$

As $\partial L_2^2 / \partial s_2 > 0$ (and/or $\partial L_2^2 / \partial s_2 < 0$), for $s_2 > (\prec) 0$, $L_2^2 > (\prec) L_2^0$. On average:

$$\partial E[L_2]/\partial s_2 = q_2 L_2^2 / \partial s_2 + (1 − q_2) \partial L_2^0 / \partial s_2$$

$$= q_2 [W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2)] − 1 /[W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2)]$$

(30)

$$+ W_2^0(L_2^2)$$

For $s_2 > (\prec) 0$, $L_2^2 > (\prec) L_2^0$ and $\partial E[L_2]/\partial s_2 > (\prec) 0$ if $W_2^1(L^1 − L_2^0) + W_2^2(L_2^0)$ rises with $L$. That requires that $W_2^0(L_2^0) > W_2^0(L_1^0) = E[L_2]$. Increases with uncertainty in region 2 (rises with $s_2$ if positive, decreases if negative) if the inverse demand in region 2 is more convex than the first one.

Overall, no unemployment is ever generated in the economy. As for the baseline region 2’s wage:

$$\partial E[W_2]/\partial s_2 = E[W_1]/\partial s_2 = q_2 W_2^1 / \partial s_2 + (1 − q_2) W_2^0 / \partial s_2$$

$$= [q_2 W^2(L_2^2) / \partial s_2 + (1 − q_2) W_2^2(L_2^0)] / \partial s_2$$

$$= [q_2 W^2(L_2^2) / \partial s_2 + (1 − q_2) W_2^2(L_2^0)] / \partial s_2$$

$$= q_2 [W^1(L_2^2)^2 / [W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2)]$$

$$− W_2^2(L_2^2) / [W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2)] + 1 − 1 /[W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2)]$$

(31)

$$= q_1 [1/[W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2) + 1] − 1 /[W^1(L^1 − L_2^2)^2 + W_2^2(L_2^2) + 1]]$$
For $s_2 > (\cdot <) L_2, E[W_2]/\partial s_2 > (\cdot <) 0$ if $W^2(L)/\partial W^2(L')$ decreases with $L$ or, equivalently, $W^2(L')/\partial W^2(L')$ decreases with $L$. That requires that $W^2(L)/W^2(L') + W^2(L'; L; - L')/W^2(L'; L; - L') < 0$. $E[W_2]$ increases with uncertainty in region 2 (rises with $s_2$ if positive, decreases if negative) when the sum of the symmetric of the Arrow-Pratt measures of risk aversion of the inverse demands (negatively sloped) is negative (this occurs if both inverse - and direct, once they are negatively sloped - demands are convex). It decreases when that sum is positive.

Consider now perfectly and positively correlated uncertainty in the two regions - case 2.b. Then, either $s$ is observed – which occurs with probability $q$ – and

$$L^1(W_2 + a - s) + L^1(W_2 - s) = L^1.$$ 

Or, with probability $(1 - q)$, $-\frac{qs}{1 - q}$ is and:

$$L^1(W_2 + a + \frac{qs}{1 - q}) + L^2(W_2 + \frac{qs}{1 - q}) = L^1.$$ 

Then:

$$\partial L^1/\partial s = 1$$

$$\partial L^2/\partial s = -\frac{q}{1 - q}$$

$$\partial E[W_2]/\partial s = q\partial L^1/\partial s + (1 - q)\partial L^2/\partial s = 0$$

Employment is always stabilized after a shock – local employment remains constant after a realization of $Y$. Hence:

$$\partial E[L^1(W_2 + a - Y)]/\partial s = -\partial E[L^2(W_2 - Y)]/\partial s = 0$$

With negatively correlated uncertainty – hypothesis 2.c., either $s$ is observed – which occurs with probability $q$ – and:

$$L^1(W_2 + a - s) + L^2(W_2 + s) = L^1$$

or

$$W^1(L_2) - a + s = W^2(L; - L_2) - s$$

or $-\frac{qs}{1 - q}$ is observed – which occurs with probability $(1 - q)$ – and:

$$W^1(L_2) - a - \frac{qs}{1 - q} = W^2(L; - L_2) + a + \frac{qs}{1 - q}$$

Obviously, the multipliers double with respect to unilateral uncertainty surrounding only one region and the consequences of a rise in uncertainty are the same: $E[L_1]$ increases with uncertainty (rises with $s$ if positive, decreases if negative) when the inverse demand of region 1 is more convex than that of region 2.

$E[W_2]$ and $E[W_1]$ increase with uncertainty (rises with $s$ if positive, decreases if negative) when the sum of the symmetric of the Arrow-Pratt measures of risk aversion of the inverse demands (negatively sloped) is negative (this occurs if both inverse - and direct, once they are negatively sloped - demands are convex). Summarizing:

**Proposition 4:** Under free market, productivity uncertainty and ex-post location decisions:

**4.1.** A unilateral or generalized and negatively correlated across regions increase in risk will result in a rise (decrease) in the expected employment of the region with more convex (concave) inverse labor demand. Expected wages will rise (decrease) with such uncertainty if the sum of relative concavity (measured by the symmetric of the Arrow-Pratt measure of risk aversion) of the inverse demand functions is negative (positive).

**4.2.** Perfectly and positively correlated wage shocks will result in equal wage movements, neutralizing any effect on the expected wages or local labor force of the economy.

### 3.2.2. Ex-ante location arrangements

Productivity dispersion has no effect on the expected equilibrium if ex-ante decisions regarding quantities must be made.

**Proposition 5:** Under free market, productivity uncertainty, and ex-ante arrangements, labor market outcomes are invariant to dispersion in the wage distribution.

### 4. Partial coverage - The perfect mobility case

Assume 3.a, 4.b, and 5.a. of section I. The wage in the two regions differ potentially by more than $a$. In region 1, the wage is fixed at level $W_1$. As the other region’s wage is fixed, it will decrease till all the labor force is employed.

Under constrained wages in sector 1 and free mobility, expected wage utilization may converge, therefore, never be fulfilled. Then the compensating differential does not have a specific meaning – we can consider it added to form the institutional $W_1$, or that the resulting equilibrium wage differential is already sufficient to compensate for it – i.e., naturally larger than $a$: the equilibrium wage of region 2, $W_2 < W_1 - a$ for the minimum wage to be a binding restriction. Otherwise, we fall into the case of section II.

**Proposition 6:** In a dualistic model with perfect mobility and institutional wage fixed in one of the sectors, the equilibrium wage in the second sector is lower than the free market wage. There will be no unemployment.

**6.2.** An increase in local externalities favouring one of the regions will have neither effect or render the institutional wage barrier inactive.

### 4.1. Quantity uncertainty

**4.1.1. Ex-post flexibility**

Consider case 2.a: only sector i is affected by uncertainty.

Either $s_i$ is observed – which occurs with probability $q_i$ – and

$$L^1(W_1) + L^2(W_2) + s_i = L_i$$

Then:

$$\partial L^1_i/\partial s_i = -1/L_i(W_2)^{-} > 0$$

Or, with probability $(1 - q_i)$, $-\frac{q_i s_i}{1 - q_i}$ is and:

$$L^1(W_1) - \frac{q_i s_i}{1 - q_i} + L^2(W_2) = L_i$$

and

$$\partial L^2_i/\partial s_i = \frac{q_i s_i}{1 - q_i}/L_i(W_2)^{-} < 0$$

As $\partial L^1_i/\partial s_i > 0$ (and/or $\partial L^2_i/\partial s_i < 0$), for $s_i > (\cdot <) 0$, $W_2 > (\cdot <) W_2^0$. On average:

$$\partial E[W_2]/\partial s_i = q_i \partial L^1_i/\partial s_i + (1 - q_i) \partial L^2_i/\partial s_i$$

$$= q_i [1/L_i(W_2)^{-} - 1/L_i(W_2)]$$

$$= q_i [L_i(W_2)^{-} - L_i(W_2)]/L_i(W_2)^{-}L_i(W_2)$$

For $s_i > (\cdot <) 0$, $W_2 > (\cdot <) W_2^0$ and $\partial E[W_2]/\partial s_i > (\cdot <) 0$ if $L_i(W_2)^{-}$ rises with $W$. That occurs when the uncovered sector labor demand is convex – that is, $L_i(W_2)^{-} > 0$. $E[W_2]$ decreases with uncertainty (rises with $s_i$ if positive, decreases if negative) when the uncovered region labor demand is concave.

Overall, no unemployment is ever generated in the economy. Labor force allocation changes only aftershocks in sector 1 - and by the same magnitude. As $E[X_i] = 0$, a rise in $s_i$ has no effect on expected local employment:

$$\partial E[L^1(W_1) + X_1]/\partial s_1 = -\partial E[L^2(W_2)]/\partial s_1 = 0$$

(35)
and:
\[ \partial E[L^1(W_1)]/\partial s_2 = -\partial E[L^2(W_2) + X_2]/\partial s_2 = 0 \] (36)

Consider now perfectly and positively correlated uncertainty in the two regions - case 2b. Then, either \( s \) is observed - which occurs with probability \( q \) - and
\[ L^1(W_1) + s + L^2(W_2) + s = L^1; \]
\[ L^1(W_1) - \frac{q s}{1 - q} + L^2(W_2) = L^1; \]

Multiplicities will double relative to the unilateral uncertainty case - and conclusions remain unaltered.

As for employment, again:
\[ \partial E[L^1(W_1) + X]/\partial s = -\partial E[L^2(W_2) + X]/\partial s = 0 \] (37)

With negatively correlated uncertainty - hypothesis 2.c. -, the wage is invariant to and stabilized at the level for which:
\[ L^1(W_1) + L^2(W_2) = L^1; \]

Employment in each region fluctuates mimicking the local shock. Expected local employment does not vary with uncertainty.

Summarizing:
**Proposition 7**: In a dualistic model with perfect mobility and institutional wage fixed in one of the regions, under quantity uncertainty and ex-post location flexibility:

- **7.1.** A unilateral or generalized and positively correlated across regions increase in risk will result in a rise (decrease) in the expected "net" wage if the uncovered region labor demand is concave (concave).
- **7.2.** A unilateral or generalized and perfectly correlated across regions increase in risk will have no effect on expected local employment.
- **7.3.** Perfectly and negatively correlated labor demand shocks will result in equal employment movements, neutralizing any effect on the expected wages in the economy.

4.1.2. **Ex-ante location arrangements**

Ex-ante location arrangements would be problematic to model under perfect mobility when uncertainty in covered region 1 is staged: unemployment would necessarily be generated. If we consider this a possibility, and allow for (un)correlated uncertainty also in region 2, the general equilibrium for \( s_2 > 0 \) can be written as:
\[ W_1[l(q_1 + (1 - q_1)[L^1(W_1) - \frac{q_1 s_1}{1 - q_1}]/L_1)] = q_2 W^2(L^1 - L_1 - s_2) + (1 - q_2) W^2(L^1 - L_1 + \frac{q_2 s_2}{1 - q_2}) \]

Yet, if there is uncertainty in region 2, it may be worthwhile to face unemployment chances in region 1 even when the shock is positive:
\[ W_1[l(L^1(W_1) + s_1 L_1 + (1 - q_1)[L^1(W_1) - \frac{q_1 s_1}{1 - q_1}]/L_1) = W_1 L^1(W_1)/l_1 = q_2 W^2(L^1 - L_1 - s_2) + (1 - q_2) W^2(L^1 - L_1 + \frac{q_2 s_2}{1 - q_2}) \]

Then we fall into the imperfect mobility environment of the next Section, IV. We therefore discard the possibility, consider case 2a, and that only sector 2 is affected by uncertainty. Always:
\[ L_1 = L^1(W_1) \]

and
\[ E[W_2] = q_2 W^2(L^1 - L^1(W_1) - s_2) + (1 - q_2) W^2(L^1 - L^1(W_1) + \frac{q_2 s_2}{1 - q_2}) \]

\[ \partial E[W_2]/\partial s_2 = q_2 W^2[L^1 - L^1(W_1) + \frac{q_2 s_2}{1 - q_2}] \]

\[ W^2[1 - L^1(W_1) - s_2] \]

The expected wage in region 2 rises (decreases) with local uncertainty - \( \partial E[W_2]/\partial s_2 > 0 \) when \( s_2 > 0 \), \( \partial E[W_2]/\partial s_2 < 0 \) when \( s_2 < 0 \). This occurs because labor demand is negatively sloped, when \( W^2(L^1) > 0 \) i.e., when labor demand in region 2 is convex (concave).

As \( L^1(W_1) \) is fixed, also is the labor force. The effect of a rise in its volatility is nil:
\[ \partial E[L_1]/\partial s_2 = \partial E[L^1(W_1) + X_1]/\partial s_2 = 0 \] (39)

**Proposition 8**: In a dualistic model with perfect mobility and institutional wage fixed in one of the regions, subject to uncertainty and to ex-ante decisions:

- **8.1.** A unilateral increase in risk in the uncovered sector will result in a rise (decrease) in the expected "net" wage if the uncovered sector labor demand is convex (concave).
- **8.2.** An increase in risk in the covered region will lead to an imperfect mobility stage.

4.2. **Productivity uncertainty**

4.2.1. **Ex-post flexibility**

Consider case 2a: only one region is affected by uncertainty. Conclusions differ according to which region is affected by \( Y \).

Either \( s_1 \) is observed - which occurs with probability \( q_1 \) and
\[ L^1(W_1 - s_1) + L^2(W_2) = L^1; \] or \( W^2 = W^2[L^1 - L^1(W_1 - s_1)] \]

Then:
\[ \partial W^2/\partial s_1 = L^1(W_1 - s_1)/L^1(W_2) \]
\[ = L^1(W_1 - s_1) W^2[L^1 - L^1(W_1 - s_1)]' > 0 \]

Or, with probability \( (1 - q_1) \), \( q_1 s_1 \) is and:
\[ L^1(W_1 + \frac{q_1 s_1}{1 - q_1}) + L^2(W_2) = L^1; \] or \( W^2 = W^2[L^1 - L^1(W_1 + \frac{q_1 s_1}{1 - q_1})] \]

and
\[ \partial W^2/\partial s_1 = -\frac{q_1}{1 - q_1} \]
\[ = -\frac{q_1}{1 - q_1} L^1(W_1 + \frac{q_1 s_1}{1 - q_1}) W^2[L^1 - L^1(W_1 + \frac{q_1 s_1}{1 - q_1})]' < 0 \]

As \( \partial W^2/\partial s_1 > 0 \) (and/or \( \partial W^2/\partial s_1 < 0 \)), for \( s_1 > (> )0 \), \( W^2 > (>) W^2 \). On average:
\[ \partial E[W_2]/\partial s_1 = q_1 \partial W^2/\partial s_1 + (1 - q_1) \partial W^2/\partial s_1 \]
\[ = q_1 L^1(W_1 - s_1) / L^1(W_2) L^1(W_1 + \frac{q_1 s_1}{1 - q_1})' / L^1(W_2) \]
\[ = q_1 [L^1(W_1 - s_1) W^2[L^1 - L^1(W_1 - s_1)]' \]
\[ - L^1(W_1 + \frac{q_1 s_1}{1 - q_1}) W^2[L^1 - L^1(W_1 + \frac{q_1 s_1}{1 - q_1})]' \] (40)
If it becomes worthwhile to face unemployment chances in sector 1 even when the shock is positive:

\[ W_1\{L^1(W_1 - s_1)q_1 + (1 - q_1)L^1(W_1 + \frac{q_1s_1}{1 - q_1})\}/L_1 = W^2(L^- - L_1) \]

Then we fall into the imperfect mobility environment of the next section, IV.

Consider then case 2.a and that only region 2 is affected by uncertainty. As

\[ L_1 = L^1(W_1) \]

productivity uncertainty has no effect on expected outcomes, once it only affects the uncovered region wage and in an additive fashion.

**Proposition 10:** In a dualistic model with perfect mobility and institutional wage fixed in one of the regions, subject to productivity uncertainty and to ex-ante decisions:

10.1. A unilateral increase in risk in the uncovered region will have no effect on employment allocation or expected wage.
10.2. An increase in risk in the covered region will lead to an imperfect mobility stage.

5. Partial coverage and imperfect mobility - The Harris-Todaro model

Assume 3.b, 4.b and 5.a of section I. The wage in the two regions differs. In region 1, the net wage is fixed at level \( W_1 \).

As region 2’s wage is free, it will decrease till all the local labor force is employed:

\( L^-_2 = L^2(W_2) \)

However, to have access to wage \( W_1 \), people have to locate there, or to specialize if we are addressing industry rather than regional affiliation – implying that unemployment will be generated in the region. There will be labor force flows till

\[ W_1 \times \text{Probability of Employment in region 1} = W_2 + a \]

That is, in the long run we expect that:

\[ W_1L^1(W_1)/L^-_1 = W_1L_1(W_1)/[L^- - L^2(W_2)] = W_2 + a \quad (44) \]

\[ W_1L_1(W_1) = (W_2 + a)[L^- - L_2(W_2)] \]

\[ = [W_2(L^- - L^-_1) + a]L^-_1 \]

\[ \partial W_2/\partial a = -[L^- - L^2(W_2)]/[L^- - L^2(W_2) - (W_2 + a)L^2(W_2)] < 0 \quad (45) \]

\[ \partial L^-_1/\partial a = -L^-_1/[W^2(L^- - L^-_1) + a - L^-_1W^2(L^- - L^-_1)] < 0 \quad (46) \]

\[ \partial[W_1L_1 + W_2L_2]/\partial a = [L^2(W_2) + W_2L^2(W_2)]/W_2/\partial a \quad (47) \]

the wage bill – and the expected wage in the economy – will increase (decrease) with a if the absolute value of the elasticity of labor demand in region 2 is larger than 1. (Aggregate workers' welfare, \( W_1L_1 + (W_2 + a)L_2 = (W_2 + a)L^- \), will always increase with a.)

**Proposition 11:** 11.1. Consider a dualistic model with imperfect mobility in the short-run. The equilibrium wage in the second sector may be higher or lower than the free market equilibrium, and there will be unemployment in the institutional sector or urban region.
11.2. An increase in local externalities favouring the uncovered (covered) region will attract the labor force to the latter and depress the local (raise the uncovered regions) wage. The average wage in the economy will increase (decrease) if the absolute value of the elasticity of labor demand in covered region exceeds (is smaller than) 1.

5.1. Quantity uncertainty

5.1.1. Ex-post flexibility

Let us consider 2.a. Assume $X_2 = 0$: uncertainty only affects covered region 1's employment. Then:

Either $s_1$ is observed – which occurs with probability $q_1$ – and

$$W_1[L^1(W_1) + s_1] = (W_2^a + a)[L_1^2(W_2^a)]$$

$$= [W_2^a(L_1^2 - L_1^2) + a]L_1^2$$

$$\frac{\partial q_1}{\partial s_1} = W_1/[W_2^a(L_1^2 - L_1^2)] + a - W_2^a(L_1^2 - L_1^2)L_1^2 > 0$$

$$\partial q_1^a/\partial s_1 = W_1/[L_1^2(L_1^2 - L_1^2) - (W_2^a + a)L_1^2L_1^2] > 0$$

Or, with probability (1 – $q_1$), – $\frac{\partial q_1}{\partial s_1}$ is and:

$$W_1[L^1(W_1) - \frac{q_1 s_1}{1 - q_1}] = (W_2^a + a)[L_1^2(W_2^a)]$$

$$= [W_2^a(L_1^2 - L_1^2) + a]L_1^2$$

$$\frac{\partial q_1}{\partial s_1} = \frac{- q_1}{1 - q_1}W_1/[W_2^a(L_1^2 - L_1^2)] + a - W_2^a(L_1^2 - L_1^2)L_1^2 > 0$$

$$\partial q_1^a/\partial s_1 = \frac{- q_1}{1 - q_1}W_1/[L_1^2(L_1^2 - L_1^2) - (W_2^a + a)L_1^2L_1^2] < 0$$

As $\partial q_1^a/\partial s_1 > 0$ (and/or $\partial q_1^a/\partial s_1 < 0$), for $s_1 > (<) 0$, $W_2^a > (<) 0$. On average:

$$\partial q_1^a/\partial s_1 = q_1 \frac{\partial q_1^a/\partial s_1}{\partial q_1/\partial s_1} + (1 - q_1)\frac{\partial q_1}{\partial s_1}$$

$$= q_1 [(L_1^2(W_2^a) + (W_2^a + a)L_1^2(W_2^a) - [L_1^2(W_2^a)] + (W_2^a + a)L_1^2L_1^2)$$

$$/[L_1^2(W_2^a) + aL_1^2(W_2^a)][L_1^2(L_1^2 - L_1^2) - (W_2^a + a)L_1^2L_1^2]] > 0$$

For $s_1 > (<) 0$, $W_2^a > (<) 0$ and $\partial q_1^a/\partial s_1 > (<) 0$ if $L_1^2(W_2^a + (W_2^a + a)L_1^2W_2^a)$ rises with $W$. Around $a = 0$, that occurs when the uncovered region wage bill function, $[W_2^a(W_2^a)]$, is convex in $W$. $E[W_2^a]$ decreases with uncertainty (rises with $s_1$ if positive, decreases if negative) when that ratio is concave. As $\partial q_1^a/\partial s_2 < 0$ (and/or $\partial q_1^a/\partial s_2 > 0$), for $s_2 > (<) 0$, $W_2^a > (<) 0$. On average:

$$\partial q_1^a/\partial s_2 = q_2 \frac{\partial q_1^a/\partial s_2}{\partial q_1/\partial s_2} + (1 - q_2)\frac{\partial q_1}{\partial s_2}$$

$$= q_2 [((W_2^a - W_1^1)W_1^1(W_2^a) - [L_1^2(W_2^a)] - W_4^1(W_2^a))$$

$$/[W_1^1(W_2^a)W_1^1(W_2^a) - L_1^2(W_2^a)] > 0$$

For $s_2 > (<) 0$, $W_2^a > (<) 0$ and $\partial q_1^a/\partial s_2 > (<) 0$ if $L_1^2(W_2^a - W_1^1)W_1^1(W_2^a)$ rises with $W$. That occurs when the total wage bill divided by the uncovered region's wage is convex in it – that is, if $d^2([W_1^1(L_1)^2 + W_2^a(W_2^a)]W_1^1(L_1)^2/W_1^1(W_2^a)]/W_2^a > 0$. $E[W_2^a]$ decreases with uncertainty (rises with $s_2$ if positive, decreases if negative) when that ratio is concave.
For $s_2 > 0$, $W_2^A > 0$, $W_2^B$ and, around $a = 0$, $\partial E[L_2]/\partial s_2 > 0$ if $L_2^W W_2^A$ rises (decreases) with $W$. 

Taking 2.b., and allowing for perfectly and positively correlated uncertainty we derive that:

Either $s$ is observed -- which occurs with probability $q$ -- and

$W_1 [L^1(W_1) + s] = (W_2^A + a) [L^1(L_1^W) + s]$

$= W_2^A [L^1(L_1^W) + s] + a [L^1(L_1^W)]$

Then:

$\partial W_1^B / \partial s_1 = (W_1 + W_2^B + a) [L^1(L_1^W)]$

$s - (W_2^B + a) L^2(W_2^B) > 0$

$\partial L_1^W / \partial s_1 = [W_1 + W_2^B (L_1^W - L_1^{W_1} A - s) L_1^W A]$

$W_2^B [L^1(L_1^W) - L_1^{W_1} A - s]$

Or, with probability $(1 - q)$, $-q/W_1^A$ is and:

$W_1 [L^1(W_1) - q/W_1^A] = (W_2^A + a) [L^1(L_1^W) + q/W_1^A]$

$= W_2^A [L^1(L_1^W) + q/W_1^A] + a [L^1(L_1^W)]$

$\partial W_1^B / \partial s_1 = -q/W_1^A (W_1 + W_2^B + a) [L^1(L_1^W)]$

$s - (W_2^B + a) L^2(W_2^B) < 0$

$\partial L_1^W / \partial s_1 = -q/W_1^A [W_1 + W_2^B (L_1^W - L_1^{W_1} A - s) L_1^W A]$

$W_2^B [L^1(L_1^W) - L_1^{W_1} A - s]$

As $\partial W_1^B / \partial s_1 > 0$ (and/or $\partial W_1^B / \partial s_1 < 0$, for $s > (1 - q), W_2^B > (1 - q)$). On average:

$\partial E[L_2^1]/\partial s_1 = q \partial W_1^A / \partial s_1 + (1 - q) \partial W_2^B / \partial s_1$

$q = q [W_1 + W_2^B (L_1^W - L_1^{W_1} A - s)]/ [W_2^B (L_1^W - L_1^{W_1} A - s)]$

$s - (W_2^B + a) L^2(W_2^B)

For $s > (1 - q), W_2^A > (1 - q)W_2^B$ and $\partial E[L_2^1]/\partial s_1 > (1 - q)\partial W_2^B / \partial s_1$ if but not only if $[L^2(W_1)+W_2^B W_2^B]W_2^B$ rises with $W$. Around $a = 0$, that occurs $- E[L_2^1]$ rises with uncertainty - when the uncovered region wage bill function, $[W_2^B][W_2^B]$, is convex.

For $s > (1 - q), W_2^A > (1 - q)W_2^B$ and $\partial E[L_2^1]/\partial s_1 > (1 - q)\partial W_2^B / \partial s_1$ if but not only if $[W_2^B W_2^B]W_2^B$ is rises with $W$. That occurs, and $E[L_2^1]$ decreases with uncertainty when the total wage bill divided by the uncovered region's wage is concave in it - that is, if $d^2[\partial W_1^1(W_1) + W_2^B(W_1)]/ W_1^2 < 0$. 

On average:

$\partial E[L_2^1]/\partial s_1 = q \partial W_1^A / \partial s_1 + (1 - q) \partial W_2^B / \partial s_1$

$q = q [W_1 + W_2^B (L_1^W - L_1^{W_1} A - s)]/ [W_2^B (L_1^W - L_1^{W_1} A - s)]$

$s - (W_2^B + a) L^2(W_2^B)

For $s > (1 - q), W_2^A > (1 - q)W_2^B$ and $\partial E[L_2^1]/\partial s_1 > (1 - q)\partial W_2^B / \partial s_1$ if but not only if $[L^2(W_1)+W_2^B W_2^B]W_2^B$ rises with $W$. Around $a = 0$, that occurs $- E[L_2^1]$ rises with uncertainty - when the uncovered region wage bill function, $[W_2^B][W_2^B]$, is convex.
As $\partial W^A_1/\partial s > 0$ (and/or $\partial W^B_2/\partial s < 0$), for $s > (\neq) 0$, $W^A_2 > (\neq) W^B_2$ on average:

$$
\text{expected wage},
$$

$$
\text{occurring in region 1 if, and only if, the wage bill is concave in region 1.}
$$

For $s > (\neq) 0$, $W^A_2 > (\neq) W^B_2$ and $\partial E[W_2]/\partial s < 0$ if but not only if $L^2(W) + (W + a) L^2(W')$ increases with $W$. Around $a = 0$, that occurs when the uncovered sector wage bill function, $[W L^2(W)]$, is concave. Or:

For $s > (\neq) 0$, $W^A_2 > (\neq) W^B_2$ and $\partial E[W_2]/\partial s < 0$ if but not only if $W^2(W) + L^2(W')$ rises with $W$. That occurs, and $E[W_2]$ decreases with uncertainty if but not only if the total wage bill divided by the uncovered region's wage is concave in it – that is, if $d^2 ([W L^1(W_1)] + W L^2(W_1)) / W) / dW^4 < 0$.

$$
\partial E[L_1^1]/\partial s = q \partial L_1^1 A / \partial s + (1 - q) \partial L_1^1 B / \partial s
$$

$$
q[(W_1 - W_1^2 - a) L^2(W_1^2) / [L_1^2 - L_1^2 A + s)]
$$

$$
+ a - W_1^2 L_1^2 - L_1^2 A + s) / L_1^2 - L_1^2 B
$$

$$
- [W_1 - W_1^2 - a) L^2(W_1^2) / [L_1^2 - L_1^2 B - q^2 L_1^2 A]
$$

$$
- a - W_1^2 L_1^2 - L_1^2 B - q^2 L_1^2 A)
$$

$$
= - q[(W_1 - W_1^2 - a) L^2(W_1^2) / [L_1^2 - L_1^2 B - q^2 L_1^2 A]
$$

$$
+ (W_1 - W_1^2 - a) L^2(W_1^2) /
$$

$$
[[(W_1^2 - a) L^2(W_1^2) / [L_1^2 - L_1^2 A + s]]
$$

$$
[(W_1^2 - a) L^2(W_1^2) / [L_1^2 - L_1^2 B + q^2 L_1^2 A]]
$$

$$
- q[(W_1 - W_1^2 - a) / [(W_1 L^1(W_1) + s)]
$$

$$
[(W_1^2 - a) L^2(W_1^2) / (W_1 + a)]
$$

$$
- a - W_1^2 L_1^2 - L_1^2 B - q^2 L_1^2 A)
$$

$$
+ (W_1 - W_1^2 - a) L^2(W_1^2) /
$$

$$
[[(W_1^2 - a) L^2(W_1^2) / [L_1^2 - L_1^2 A + s]]
$$

$$
[(W_1^2 - a) L^2(W_1^2) / [L_1^2 - L_1^2 B + q^2 L_1^2 A]]
$$

3. Hence, the uncertainty affecting region 2's wage has an impact on the labor force allocation:

$$
W_1 L^1(W_1) = q_1 W_2^2(L; L - L_1^1 - s_2) + (1 - q_1)
$$

$$
W_2^2(L; L - L_1^1 + q_2 s_2 / q_2^2) + a L_1^1
$$

Then:

$$
\partial L_1^1 / \partial s_2 = q_1 [W_2^2(L - L_1^1 - s_2) + (1 - q_1)] W_1^2(L - L_1^1 - s_2) + (1 - q_2)
$$

$$
W_2^2(L; L - L_1^1 + q_2 s_2 / q_2^2) + a L_1^1 + a
$$

For $s_2 > (\neq) 0$, $W_2^2(L - L_1^1 - s_2) > (\neq) W_2^2(L - L_1^1 + q_2 s_2 / q_2^2)$ and $\partial L_1^1 / \partial s_2 > (\neq) 0$ and $\partial L_1^1 / \partial s_2 < (\neq) 0$ if $W_2^2(L)$ decreases (rises) with $L$ when the uncovered region inverse demand is concave (convex) in $L$ and the labor demand function $L^2(W)$ is concave (convex) in $W$.

$$
\partial E[W_1]/\partial s_2 = - [W_1 L^1(W_1) / L_1^1 + a] / \partial s_2
$$

The expected wage will move in opposite direction to the labor force in the covered region.

**Proposition 13:** With multiple coverage and imperfect mobility, quantity uncertainty with ex-ante location decisions.

13.1. A unilateral increase in wage in the covered region will have no effect on employment allocation or expected wage.

13.2. An increase in the uncovered region will result in an increase (decrease) in the covered labor force and of the expected wage in the economy if the uncovered region labor demand is concave (convex).

5.2. Productivity uncertainty

5.2.1. Ex-post flexibility

In general, the equilibrium satisfies:

$$
W_1 L^1(W_1 - Y_1) = (W_2 + a)[L_1^1 - L_2^2(W_2 - Y_2)]
$$

$$
= [W_2^2(L; L - L_1^1 + Y_2 + a) L_1^1
$$

Let $Y_2 = 0$: uncertainty only afflicts covered region 1's employment. Then:
Either $s_1$ is observed – which occurs with probability $q_1$ – and

$$W_1 L^1(W_1 - s_1) = (W_2^c + a)[L_2 - L^2(W_2^c)]$$

$$= [W^2(L_2^1 - L_1^1 C) + a][L_2 - L^2(W_2^c)]$$

$$\partial L_1^1/\partial W_1 = -L^1(W_1 - s_1)/W_1/[W^2(L_2^1 - L_1^1 C)]$$

$$+ a - W^2(L_2^1 - L_1^1 C)/L_1^1 C > 0$$

$$\partial W_2^0/\partial s_1 = -W_1 L^1(W_1 - s_1)/[L_2 - L^2(W_2^c)]$$

$$- (W_2^c + a) L^2(W_2^c)' > 0$$

Or, with probability $(1 - q_1)$, $-q_1 s_1$ is and:

$$W_1 L^1(W_1 + q_1 s_1) = (W_2^c + a)[L_2 - L^2(W_2^c)]$$

$$= [W^2(L_2^1 - L_1^1 C) + a][L_2 - L^2(W_2^c)]$$

$$\partial L_1^1/\partial W_1 = q_1 / (1 - q_1) W_1 L^1(W_1 + q_1 s_1)' /$$

$$[W^2(L_2^1 - L_1^1 C) + a][L_2 - L^2(W_2^c)] < 0$$

$$\partial W_2^0/\partial s_1 = q_1 / (1 - q_1) W_1 L^1(W_1 + q_1 s_1)' /$$

$$[L_2 - L^2(W_2^c)(W_2^c + a) L^2(W_2^c)' ] < 0$$

As $\partial W_2^0/\partial s_1 > 0$ (and/or $\partial W_2^0/\partial s_1 < 0$, for $s_1 > (0)$, $W_2^c > (<) W_2$, $W_2^c > (<) W_2$. On average:

$$\partial E[W_2]/\partial s_1 = q_1 \partial W_2^c/\partial s_1 + (1 - q_1) \partial W_2^0/\partial s_1$$

$$= q_1 L_1^1 / (W_1 + q_1 s_1)' / [W^2(L_2^1 - L_1^1 C) + a] [L_2 - L^2(W_2^c)(W_2^c + a) L^2(W_2^c)]$$

$$- L^1(W_1 - s_1)' / [L_2 - L^2(W_2^c)(W_2^c + a) L^2(W_2^c)]$$

(58)

For $s_1 > (0)$, $W_2^c > (<) W_2^0$ and $\partial E[W_2]/\partial s_1 > (0)$ if $[L^2(W_2^c)(W_2^c + a) L^2(W_2^c)]$ rises with $W$ – provided $L^1(W_1)$ is not too concave. Around $a = 0$, that occurs when the uncovered region wage bill function, $[W^2(L_2^1)]$, is convex. $E[W_2]$ decreases with uncertainty (rises with $s_1$ if positive, decreases if negative) when the uncovered wage bill is concave – provided $L^1(W_1)$ is not too convex.

As $\partial L_1^1/\partial s_1 > 0$ (and/or $\partial L_1^1/\partial s_1 < 0$, for $s_1 > (0)$, $L_1^1 < (<) L_1^0, L_1^1 C > (<) L_1^0$. On average:

$$\partial E[L_1^1]/\partial s_1 = q_1 \partial L_1^1 C/\partial s_1 + (1 - q_1) \partial L_1^0 /\partial s_1$$

$$= q_1 L_1^1 / (W_1 + q_1 s_1)' / [W^2(L_2^1 - L_1^1 C) + a] [L_2 - L^2(W_2^c)(W_2^c + a) L^2(W_2^c)]$$

$$- L^1(W_1 - s_1)' / [W^2(L_2^1 - L_1^1 C) + a] [L_2 - L^2(W_2^c)(W_2^c + a) L^2(W_2^c)]$$

(59)

For $s_1 > (<) 0$, $L_1^1 C > (<) L_1^0$ and $\partial E[L_1^1]/\partial s_1 > (<) 0$ if $[W^2(L_2^1 - L_1^1 C)]$ decreases with $L_1$ or $W^2(L_2^1 - L_1^1 C)$ increases with $L$ – provided that $L_1^1 C$ is not too concave. That occurs when the uncovered region wage times the other region's employment $[W^2(L_2^1 - L_1^1 C)]$, is convex in $L$ – if $W^2(L_2^1 - L_1^1 C)$ increases in $L$. $E[L_1^1]$ decreases with uncertainty (decreases with $s_1$ if positive, increases if negative) when $[W^2(L_2^1 - L_1^1 C)]$ is concave in $L$ – if $W^2(L_2^1 - L_1^1 C)$ is concave in $L$ – provided $L^1(W_1)$ is not too convex.

Let now be region 2 suffering uncertainty and $Y_1 = 0$. For $s_2$ – which occurs with probability $q_2$ – and:

$$W_1 L^1(W_1) = (W_2^c + a)[L_2 - L^2(W_2^c - s_2)]$$

$$= [W^2(L_2^1 - L_1^1 C) + a] [L_2 - L^2(W_2^c - s_2)]$$

$$\partial L_1^1 C /\partial s_2 = -L_1^1 C /$$

$$[W^2(L_2^1 - L_1^1 C) + a - W^2(L_2^1 - L_1^1 C)] < 0$$

$$\partial W_2^0 /\partial s_2 = -(W_2^c + a) L^2(W_2^c - s_2)' /$$

$$[L_2 - L^2(W_2^c - s_2) - (W_2^c + a) L^2(W_2^c - s_2)]$$

$$= -(W_1 L^1(W_1)/L_1^1 C) \partial L_1^1 C /\partial s_2$$

$$= [W_1 L^1(W_1)/L_1^1 C] [W^2(L_2^1 - L_1^1 C) + s_2 + a]$$

$$- (W_1 L^1(W_1)/L_1^1 C) L_1^1 C$$

$$= 1/1 - W^2(L_2^1 - L_1^1 C)/L_1^1 C /\partial W_1 L^1(W_1)] > 0$$

With probability $(1 - q_2)$, $-q_0 s_2$ is observed and:

$$W_1 L^1(W_1) = (W_2^0 + a)[L_2 - L^2(W_2^0 + q_0 s_2)]$$

$$= [W^2(L_2^0 - L_1^0 D) + a][L_2 - L^2(W_2^0 + q_0 s_2)]$$

$$\partial L_1^0 C /\partial s_2 = q_0 / (1 - q_2) W_1 L^1(W_1) /$$

$$[W^2(L_2^0 - L_1^0 D) + a][L_2 - L^2(W_2^0 + q_0 s_2)]$$

$$= -q_0 / (1 - q_2) 1/1 - W^2(L_2^0 - L_1^0 D)/L_1^0 D /\partial W_1 L^1(W_1)] > 0$$

As $\partial W_2^0 /\partial s_2 > 0$ (and/or $\partial W_2^0 /\partial s_2 < 0$, for $s_2 > (<) 0$, $W_2^c > (<) W_2^0$. On average:

$$\partial E[W_2]/\partial s_2 = q_0 \partial W_2^0 /\partial s_2 + (1 - q_2) \partial W_2^0 /\partial s_2$$

$$= q_0 [(W_2^0 + a) L^2(W_2^0 + q_0 s_2)] /$$

$$[L_2 - L^2(W_2^0 + q_0 s_2) - (W_2^0 + a) L^2(W_2^0 + q_0 s_2)]$$

$$= q_0 / (1 - q_2) 1/1 - W^2(L_2^0 - L_1^0 D)/L_1^0 D /\partial W_1 L^1(W_1)] < 0$$

(60)

For $s_2 > (<) 0$, $L_1^0 C > (<) L_1^0$ and $\partial E[W_2]/\partial s_2 > (<) 0$ if $[W^2(L_2^0 - L_1^0 D)]$ decreases with $L$ – or if $[W^2(L_2^0 - L_1^0 D)]$ increases with $L$. $E[W_2]$ decreases with uncertainty (decreases with $s_2$ if positive, increases if negative) when $[W^2(L_2^0 - L_1^0 D)]$ increases with $L$ – decreases with $L$. $E[W_2]$ decreases with uncertainty (decreases with $s_2$ if positive, increases if negative) when $[W^2(L_2^0 - L_1^0 D)]$ increases with $L$ – decreases with $L$. $E[W_2]$ decreases with uncertainty (decreases with $s_2$ if positive, increases if negative) when $[W^2(L_2^0 - L_1^0 D)]$ increases with $L$.
As \( \partial L^{-1}\partial C/\partial s_2 < 0 \) (and/or \( \partial L^{-1}_1/D/\partial s_2 > 0 \)), for \( s_2 > 0 < 0 \), \( L^{-1}_1 < (>) L^{-1}_1 - D\). On average:

\[
\begin{align*}
\partial E[L^{-1}_1]/\partial s_2 &= q_a \partial L^{-1}_1/C/\partial s_2 + (1 - q_a) \partial L^{-1}_1/D/\partial s_2 \\
&= q_2/(L^{-1}_1) \partial L^{-1}_1/(\partial W(L^{-1}_1 - L^{-1}_1 - D) - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1) - L^{-1}_1) \\
&- L^{-1}_1/(\partial W^2(L^{-1}_1 - L^{-1}_1) + s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - s_2)) L^{-1}_1 \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - D) - \partial q_s_2/a + a/L^{-1}_1 - D) \\
&- \partial W(L^{-1}_1 - L^{-1}_1)) \\
&- 1/(\partial W^2(L^{-1}_1 - L^{-1}_1) + s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - D)) \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - q)) \\
&+ 1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a + a/L^{-1}_1 - q) \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a + a/L^{-1}_1 - q)) \\
&+ 1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - q)) \quad (61)
\end{align*}
\]

For \( s_2 > 0 < 0 \), \( L^{-1}_1 < (>) L^{-1}_1 - D \) and \( \partial E[L^{-1}_1]/\partial s_2 > 0 \) if \( W_1 L^{-1}_1(L^{-1}_1 - L^{-1}_1 - q)' \) decreases with \( L \). That occurs when the ratio \( [W_1 L^{-1}_1(L^{-1}_1 - L^{-1}_1 - q)' / D \) is convex in \( L \), i.e.,

\[
\begin{align*}
d(2) & \equiv [W_1 L^{-1}_1(L^{-1}_1 - L^{-1}_1 - q)' / D \) is convex in \( L \), i.e.,
\end{align*}
\]

\[
\begin{align*}
\text{or, with probability (1-q),} \quad \frac{q_2}{1-q} \text{is and:}
\end{align*}
\]

\[
\begin{align*}
&= q_2/(L^{-1}_1) \partial L^{-1}_1/(\partial W(L^{-1}_1 - L^{-1}_1 - D) - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1) - L^{-1}_1) \\
&- L^{-1}_1/(\partial W^2(L^{-1}_1 - L^{-1}_1) + s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - s_2)) L^{-1}_1 \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - D) - \partial q_s_2/a + a/L^{-1}_1 - D) \\
&- \partial W(L^{-1}_1 - L^{-1}_1)) \\
&- 1/(\partial W^2(L^{-1}_1 - L^{-1}_1) + s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - D)) \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - q)) \\
&+ 1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a + a/L^{-1}_1 - q) \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a + a/L^{-1}_1 - q)) \\
&+ 1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - q)) \quad (61)
\end{align*}
\]

Finally, for 2.c., and with perfectly negatively correlated uncertainty in the sectors we derive that:

either \( s \) is observed – which occurs with probability \( q \) – and

\[
\begin{align*}
W_1 L^{-1}_1(L^{-1}_1 - s) = (W_2 + a) [L^{-1}_1 - L^{-1}_1 - q] \\
&= [W^2(L^{-1}_1 - L^{-1}_1 - q) + s + a] L^{-1}_1 - q
\end{align*}
\]

Then:

\[
\begin{align*}
\partial W^2/\partial s = -[W_1 L^{-1}_1(L^{-1}_1 - s)' + (W_2 + a)] L^2(W_2 - s') / L^{-1}_1 - L^{-1}_1 - q) \\
&= [W^2(L^{-1}_1 - L^{-1}_1 - q) + s + a] L^{-1}_1 - q
\end{align*}
\]

\[
\begin{align*}
\partial E[L^{-1}_1]/\partial s_2 &= q_a \partial L^{-1}_1/C/\partial s_2 + (1 - q_a) \partial L^{-1}_1/D/\partial s_2 \\
&= q_2/(L^{-1}_1) \partial L^{-1}_1/(\partial W(L^{-1}_1 - L^{-1}_1 - D) - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1) - L^{-1}_1) \\
&- L^{-1}_1/(\partial W^2(L^{-1}_1 - L^{-1}_1) + s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - s_2)) L^{-1}_1 \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - D) - \partial q_s_2/a + a/L^{-1}_1 - D) \\
&- \partial W(L^{-1}_1 - L^{-1}_1)) \\
&- 1/(\partial W^2(L^{-1}_1 - L^{-1}_1) + s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - D)) \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - q)) \\
&+ 1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a + a/L^{-1}_1 - q) \\
&= q_2(1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a + a/L^{-1}_1 - q)) \\
&+ 1/(\partial W^2(L^{-1}_1 - L^{-1}_1 - q)^2 - \partial q_s_2/a - W^2(L^{-1}_1 - L^{-1}_1 - q)) \quad (61)
\end{align*}
\]

As \( \partial W^2/\partial s > 0 \) (and/or \( \partial W^2/\partial s < 0 \)), for \( s > < 0 \), \( W^2 > < 0 \). On average:

\[
\begin{align*}
\partial E[W^2]/\partial s &= q_a \partial W^2/\partial s + (1 - q_a) \partial W^2/\partial s \\
&= q_a(W_1 L^{-1}_1(W_1 + [W^2 - a]) L^2(W_2 - q) / L^{-1}_1 - L^{-1}_1 - q) \\
&- [W_1 L^{-1}_1(W_1 - s') + W_2(W_2 - a)] L^2(W_2 - s') / L^{-1}_1 - L^{-1}_1 - q)
\end{align*}
\]}
As \( \partial L_{1}^{-1}\mathcal{C}/\partial s > 0 \) (and/or \( \partial L_{1}^{-1}\mathcal{D}/\partial s < 0 \)), for \( s > (\textless 0), L_{1}^{-1}\mathcal{C} > (\textless) L_{1}^{-1}\mathcal{D} \). On average:

\[
\begin{align*}
\partial E[L_{1}^{-1}]/\partial s &= q \partial L_{1}^{-1}\mathcal{C}/\partial s + (1 - q) \partial L_{1}^{-1}\mathcal{D}/\partial s \\
&= q[(L_{1}^{-1}- W_{1}L_{1}(W_{1} - s))/\partial L_{1}^{-1}(L_{1}^{-1} - s) - W_{1}L_{1}(W_{1} + qS_{1})\mathcal{Y}]/\partial L_{1}^{-1} - W_{1}L_{1}(W_{1} - s)]/\partial L_{1}^{-1}
\end{align*}
\]

(65)

**Proposition 14**: In dualistic model with imperfect mobility in the short-run, under productivity uncertainty and end-post location flexibility:

14.1. A unilateral increase in risk in the covered region will result in a rise (decrease) in the expected “net” wage if the uncovered region’s wage bill is convex (concave). Expected local labor force increases (decreases) with uncertainty if the inverse demand of the uncovered region multiplied by excess employment relative to total supply is concave (convex) in \( L \).

14.2. A unilateral increase in risk in the uncovered region will result in a rise (decrease) in the expected wage if the slope of the uncovered region’s inverse demand multiplied by the square of the excess employment relative to total labor supply increases (decreases) with employment. Expected local labor force increases (decreases) with uncertainty if the ratio \([W_{1} L_{1}(W_{1}) - W_{2}^{2}(L_{1}^{-1} - L_{1})]/L_{1} \) is concave (convex) in \( L \).

14.3. A perfectly correlated across regions increase in risk will result in a composite of the unilateral effects – a sum if positive, a difference if negative.

5.2.2. Ex-ante location arrangements

The expected wage bill in region 2 is constant, once uncertainty is additive to the wage. Hence, only the uncertainty affecting region 1’s productivity has an impact on the labor force allocation:

\[
W_{1}[qL_{1}(W_{1} - s_{1}) + (1 - q) L_{1}(W_{1} + \frac{qS_{1}}{1 - q})] = [W^{2}(L_{1}^{-1} - L_{1})^{-1} + aL_{1}^{-1}]
\]

Then:

\[
\begin{align*}
\partial L_{1}^{-1}/\partial s_{1} &= q_{1}[L_{1}(W_{1} + \frac{q_{1}S_{1}}{1 - q_{1}}) - L_{1}(W_{1} - s_{1})]/L_{1}^{-1}
\end{align*}
\]

(66)

\[
\begin{align*}
[W^{2}(L_{1}^{-1} - L_{1})^{-1} + a - W^{2}(L_{1}^{-1} - L_{1})^{-1}]/L_{1}^{-1}
\end{align*}
\]

For \( s_{1} > (\textless) 0, L_{1}(W_{1} + \frac{q_{1}S_{1}}{1 - q_{1}})/L_{1}^{-1} > (\textless) L_{1}(W_{1} - s_{1}) \) and \( \partial L_{1}^{-1}/\partial s_{1} > (\textless) 0 \) if \( L_{1}^{-1}(W_{1}) \) increases with \( W_{1} \); \( \partial L_{1}^{-1}/\partial s_{1} \) decreases with uncertainty when the covered region labor demand is convex in \( W \). For \( s_{1} > (\textless) 0, L_{1}^{-1} > (\textless) 0 \) if the covered region labor demand is concave in \( W \).

\[
\begin{align*}
\partial E[W_{2}]/\partial s_{1} &= -\frac{W^{2}}{L_{1}^{-1} - L_{1}} \partial L_{1}^{-1}/\partial s_{2}
\end{align*}
\]

(67)

The expected wage will move in the same direction as the labor force in the covered region.

**Proposition 15**: With multiple coverage and imperfect mobility, productivity uncertainty with ex-ante location decisions.

15.1. A unilateral increase in risk in the uncovered region will have no effect on employment allocation or expected wage.

15.2. An increase in risk in the covered region will result in an increase (decrease) in the local labor force and of the expected wage in the economy if the covered region labor demand is convex (concave).

6. Multiple or global coverage under imperfect mobility - The Bhagwati-Hamada model

Assume 3.b, 4.b. and 5.b. of section I. The wages are fixed in both regions, at levels \( W_{i}, i = 1, 2 \). One can see this same (technically speaking) scenario in, for example, Bhagwati and Hamada (1974).

Equilibrium is defined by:

\[
W_{1} L_{1}(W_{1})/L_{1}^{-1} = W_{2} L^{2}(W_{2})/L_{2}^{-1} + a
\]

(68)

The average wage in the economy \([W_{1} L_{1}(W_{1}) + W_{2} L^{2}(W_{2})]/L_{1}^{-1} \) is fixed.

Differentiating (68):

\[
\begin{align*}
\partial L_{1}^{-1}/\partial a &= -(L_{1}^{-1} - L_{1})/L_{1}^{-1} \partial L_{1}^{-1}(W_{1}) \\
&+ W_{2} L^{2}(W_{2}) + a(L_{1}^{-1} - W_{2} L_{2}^{-1}) < 0
\end{align*}
\]

(69)

**Proposition 16**: 16.1. With multiple coverage and imperfect mobility, both regions will experience unemployment and expected “net” wage is equalized across regions. The average wage in the economy does not change with a local differential, once the wage bill is fixed. Also:

16.2. An increase in the local differential favouring one of the regions will attract the labor force to it. Local expected wage will decrease there and rise – as well as the expected worker’s welfare - in the other sector.

6.1. Quantity uncertainty

6.1.1. Ex-post flexibility

Then, local employment, being demand determined:

\[
L_{i} = L(W_{i}) + X_{i}
\]

Equilibrium is defined by:

\[
W_{1} L_{1}(W_{1})/L_{1}^{-1} = W_{2} L^{2}(W_{2}) + X_{2}/L_{2}^{-1} + a
\]

If \( a \equiv 0 \), one can show that the expected wage in the economy will equal the wage bill divided by \( L_{1}^{-1} \): \((W_{1} L_{1}(W_{1}) + X_{1}) + W_{2} L^{2}(W_{2}) + X_{2}) / L_{1}^{-1} \). Additive uncertainty will have no effect on the equilibrium wage.

One can re-write condition (166) as:

\[
W_{1} L_{1}(W_{1})/L_{1}^{-1} + a(L_{1}^{-1} - L_{1}) L_{1}^{-1}
\]

= \( W_{2} L^{2}(W_{2}) + X_{2}/L_{2}^{-1} + a(L_{1}^{-1} - L_{1}) L_{1}^{-1}
\]

Consider case 2.a: only region \( i \) is affected by uncertainty and \( X_{i} = 0 \). For equilibrium:

Either \( s_{1} \) is observed – which occurs with probability \( q_{1} \) – and

\[
\begin{align*}
\partial L_{1}^{-1}/\partial s_{1} &= -\partial L_{2}^{-1}/\partial s_{1} = -L_{1}^{-1}A/\partial s_{1} = L_{2}^{-1}A/\partial s_{1}
\end{align*}
\]

(68)

Or, with probability \((1 - q_{1})\), \(-q_{1}/q_{1}W_{1} \) and:

\[
\begin{align*}
\partial L_{1}^{-1}/\partial s_{1} &= -\partial L_{2}^{-1}/\partial s_{1} = -\partial L_{2}^{-1}/\partial s_{1} = -q_{1}/q_{1}W_{1}/L_{2}^{-1} W_{1}
\end{align*}
\]

(69)
As $\partial L_1^{b} / \partial s_1 > 0 (\text{and/or } \partial L_1^{a} / \partial s_1 < 0)$, for $s_1 > (\leq 0), \ L_1^{a} > (\leq) L_1^{b}$. On average:

$$\partial E[L_1^{a}] / \partial s_1 = -\partial E[L_1^{a}] / \partial \psi \partial s_1 = q_1 \partial E[L_1^{a}] / \partial s_1 + (1 - q_1) \partial E[L_1^{b}] / \partial s_1$$

$$= q_1 W_1 (L_1^{a} - W_1^{L_1^{a}} + W_1 s_1 + W_2 L_2^{L_2}(W_2) + a(L_1^{a} - 2L_1^{b}))$$

(70)

As $L_1^{a} > (\leq) L_1^{b}$ and $L_2^{a} < (\leq) L_2^{b}$ for $s_1 > (\leq 0)$, $\partial E[L_1^{a}] / \partial s_1 < (\leq) 0$ around a = 0, expected labor force flows away from the region where uncertainty rises.

Let there be case 2.b: $X_1 = X_2 = X$. In equilibrium:

Either $s$ is observed—which occurs with probability $q$ and $\partial L_1^{a} / \partial s = -\partial L_1^{a} / \partial s = (L_2^{a} W_1 - L_1^{a} W_2)/\partial s$:

$$W_1 [L_1^{a} + s] + W_2 [L_2^{L_2}(W_2) + s] + a(L_1^{a} - 2L_1^{b})$$

$$= L_2^{a} W_1 [1 - L_1^{a} (W_1) + s] / [L_2^{L_2}(W_2) + s]$$

$$+ (aL_1^{a} W_1 / [L_2^{L_2}(W_2) + s])$$

(71)

$$W_1 [L_1^{a} + s] + W_2 [L_2^{L_2}(W_2) + s] + a(L_1^{a} - 2L_1^{b})$$

$$\partial L_1^{a} / \partial s > 0$$, and, for $s > (\leq 0), L_1^{a} > (\leq) L_1^{b}$, if $L_2^{L_2}(W_2)$ is a generalized positive shock, the labor force flows away from the region of higher aggregate demand—farther away, the other becomes more appealing.

Or, with probability $(1 - q)$, the aggregate is and:

$$\partial L_1^{b} / \partial s = -\partial L_1^{b} / \partial s = -q / (1 - q) (L_2^{b} W_1 - L_1^{b} W_2)/\partial s$$

$$W_1 [L_1^{b} (W_1) + s] + W_2 [L_2^{L_2}(W_2) + s] + a(L_1^{a} - 2L_1^{b})$$

$$= L_2^{b} W_1 [1 - L_1^{b} (W_1) - q / (1 - q) / [L_2^{L_2}(W_2) + s]$$

$$+ (aL_1^{b} W_1 / [L_2^{L_2}(W_2) - q / (1 - q))$$

$$W_1 [L_1^{b} (W_1) + s] + W_2 [L_2^{L_2}(W_2) + s] + a(L_1^{a} - 2L_1^{b})$$

Then:

$$\partial E[L_1^{b}] / \partial s = -\partial E[L_1^{b}] / \partial s = q \partial E[L_1^{a}] / \partial s + (1 - q) \partial E[L_1^{b}] / \partial s$$

$$= q((L_2^{b} W_1 + L_1^{b} W_2)/\partial s$$

$$W_1 [L_1^{b} (W_1) + s] + W_2 [L_2^{L_2}(W_2) + s] + a(L_1^{a} - 2L_1^{b})$$

$$= L_2^{b} W_1 (1 - L_1^{b} (W_1) - q / (1 - q) / [L_2^{L_2}(W_2) - q / (1 - q]$$

$$+ (aL_1^{b} W_1 / [L_2^{L_2}(W_2) - q / (1 - q))$$

$$W_1 [L_1^{b} (W_1) + s] + W_2 [L_2^{L_2}(W_2) + s] + a(L_1^{a} - 2L_1^{b})$$

(72)

If $W_1 > W_2$, one can show that—a around $a = 0 - L_1^{a} / [L_2^{L_2}(W_2) - s]$ decreases with $s$. Then, for $s > 0$, $\partial E[L_1^{a}] / \partial s < 0$; for $s < 0$, $\partial E[L_1^{a}] / \partial s > 0$.

Proposition 17: With multiple coverage and imperfect mobility, under quantity uncertainty and ex-post location flexibility:

17.1. The expected (or average) wage in the economy does not change with uncertainty.

17.2. A unilateral increase in risk will result in a flight of labor force out of the inflicted region.
17.3. A generalized and positively correlated across regions increase in risk will result in a flow of labor force out of the region with smaller aggregate employment.

17.4. A rise in perfectly and negatively correlated labor demand uncertainty will result in a flow of labor force out of the region with highest wage.

6.1.2. Ex-ante location arrangements

Proposition 18: With multiple coverage and imperfect mobility, quantity uncertainty with ex-ante location decisions render labor force allocation and expected wages invariant to uncertainty.

6.2. Productivity uncertainty

6.2.1. Ex-post flexibility

Then, local employment, being demand determined:

\[ L_i = L_i(W_i - Y_i) \]

Equilibrium is defined by:

\[ W_i L_i(W_1 - Y_1)/L_i = W_2 L_2(W_2 - Y_2)/L_2 + a \quad (73) \]

One can re-write condition (73) as:

\[ W_i L_i(W_1 - Y_1)(L_1^{-1} - L_1^{-1}) = W_2 L_2(W_2 - Y_2)(L_1^{-1} + a(L_1^{-1} - L_1^{-1})) \]

As for \( a > 0 \), the expected wages in the economy equals the wage bill divided by \( L_1^{-1} \) and \( L_2^{-1} \). The expected wage in the economy rises with uncertainty if at least one of the demands of the argument of the random is sufficiently convex.

Consider case 2.a: only region 1 is affected by uncertainty and \( Y_2 = 0 \). For equilibrium:

Either \( s_1 \) is observed – which occurs with probability \( q_1 \) and

\[ \partial L_i^{-1} / \partial s_1 = -\partial L_1^{-1} / \partial s_1 = -L_i^{-1} W_i L_i(W_1 - s_1) \]

\[ [W_i L_i(W_1 - Y_1) + W_2 L_2(W_2 - Y_2)] \]

\[ (L_1^{-1} - L_1^{-1}) ] > 0 \]

Or, with probability \((1 - q_1)\), \( q_1 \) and:

\[ \partial L_1^{-1} D / \partial s_1 = -\partial L_1^{-1} D / \partial s_1 = \frac{q_1}{1 - q_1} - W_i L_i(W_1 + \frac{q_1}{1 - q_1}) \]

\[ [W_i L_i(W_1 + \frac{q_1}{1 - q_1}) + W_2 L_2(W_2 - \frac{q_1}{1 - q_1})] \]

\[ (L_1^{-1} - L_1^{-1}) ] < 0 \]

As \( \partial L_i^{-1} / \partial s_1 > 0 \) (and/or \( \partial L_1^{-1} D / \partial s_1 < 0 \) for \( s_1 > (\frac{1}{q_1} - 1) \), \( L_1^{-1} > (\frac{1}{q_1} - 1) \).

Average:

\[ \partial E[L_i^{-1}]/\partial s_1 = -\partial E[L_i^{-1}]/\partial s_1 \]

\[ q_1 \partial E[L_i^{-1}]/\partial s_1 + (1 - q_1) \partial E[L_1^{-1} D]/\partial s_1 \]

\[ q_1 W_i [L_1^{-1} D L_i(W_1 + \frac{q_1}{1 - q_1})] \]

\[ [W_i L_i(W_1 + \frac{q_1}{1 - q_1}) + W_2 L_2(W_2) + a(L_1^{-1} - 2L_1^{-1} D)] \]

\[ -L_2^{-1} C L_1(W_1 - s_1)/[W_i L_i(W_1 - s_1) + W_2 L_2(W_2) + a(L_1^{-1} - 2L_1^{-1} C)] \]

Expected labor force flows away from the region where uncertainty rises, provided local labor demand is not too convex.

Let there be case 2.b: \( Y_1 = Y_2 = Y \). In equilibrium:

Either \( s_1 \) is observed – which occurs with probability \( q \) and

\[ \partial L_i^{-1} C / \partial s_1 = -\partial L_2^{-1} C / \partial s_1 = -[L_2^{-1} C W_1 L_i(W_1 - s_1)] \]

\[ -L_2^{-1} C W_2 L_2(W_2 - s)] \]

\[ W_i L_i(W_1 - s_1) + W_2 L_2(W_2 - s) + a(L_1^{-1} - 2L_1^{-1} C)] \]

\[ -L_2^{-1} C W_1 L_i(W_1 - s_1) - L_1^{-1} W_2 L_2(W_2 - s) ] \]

\[ (L_2^{-1} C W_1 L_i(W_1 - s_1) - a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ -L_2^{-1} C W_1 L_i(W_1 - s_1) - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ (L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ -L_2^{-1} C W_1 L_i(W_1 - s_1) - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ (L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ -L_2^{-1} C W_1 L_i(W_1 - s_1) - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ (L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ -L_2^{-1} C W_1 L_i(W_1 - s_1) - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ (L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ -L_2^{-1} C W_1 L_i(W_1 - s_1) - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ (L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ -L_2^{-1} C W_1 L_i(W_1 - s_1) - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ (L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ -L_2^{-1} C W_1 L_i(W_1 - s_1) - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]

\[ (L_1^{-1} W_1 - s_1 - (a(L_1^{-1} C W_1 L_i)] + W_2 L_2(W_2 - s)] \]
only if, $L^1(W_1)'$ and $L^1(W_1)$ increases with the argument less rapidly than $L^2(W_2)'$ does. When $L^1(W_1 - s)'$ and $L^1(W_1 - s)$, $E(L_{i-1})$ increases with $s > 0$ if $L^1(W_1)'$ and $L^1(W_1)$ increases with the argument more rapidly than $L^2(W_2)'$ does.

Let there be case 2.c: $Y_i = -Y_j = -Y$. In equilibrium:

Either $s$ is observed – which occurs with probability $q$ – and

$$
\partial L_{i-1}^C/\partial s = -\partial L_{i-2}^D/\partial s = -L^2 C(W_1(W_1 - s)) + L^1 C(W_2(W_2 + s)) \leq 0
$$

Or, with probability $(1 - q)$, $-\frac{q}{1 - q}$ is and:

$$
\partial L_{i-1}^D/\partial s = -\partial L_{i-2}^D/\partial s = \frac{q}{1 - q}
$$

$$
[L^2 C(W_1(W_1 + \frac{q s}{1 - q} + L^1 D(W_2(W_2 - \frac{q s}{1 - q})))]
$$

$$
[L^1 C(W_1 + \frac{q s}{1 - q}) + L^2 C(W_2(W_2 + \frac{q s}{1 - q}) + a[L^1 - 2 L^1 D])] < 0
$$

As $\partial L_{i-2}^C/\partial s > 0$ (and $\partial L_{i-1}^D/\partial s < 0$), for $s > (0), L_{i-2}^C > (0), L_{i-1}^D$. Then:

$$
\partial E[L^1 C]/\partial s = q \partial E[L^1 C]/\partial s + (1 - q) \partial E[L^1 D]/\partial s = q[L^2 D(W_1(W_1 + \frac{q s}{1 - q} + L^1 D(W_2(W_2 - \frac{q s}{1 - q})))]
$$

$$
[L^1 C(W_1 + \frac{q s}{1 - q}) + L^2 C(W_2(W_2 + \frac{q s}{1 - q}) + a[L^1 - 2 L^1 D])]
$$

$$
-L^2 C(W_1(W_1 - s)) + L^1 C(W_2(W_2 + s))
$$

$$
[L^1 C(W_1(W_1 + \frac{q s}{1 - q}) + L^2 C(W_2(W_2 + \frac{q s}{1 - q}) + a[L^1 - 2 L^1 D])]/L^2 (W_2 + s)
$$

$$
+
\frac{q s}{1 - q})/L^2 (W_2 + \frac{q s}{1 - q}) - (a L^1 D/W_1)
$$

$$
+ L^2 (W_2 - \frac{q s}{1 - q})/L^2 (W_2 + \frac{q s}{1 - q}) - (a L^1 D/W_1)
$$

$$
\partial L_{i-1}^D/\partial s = q[L^1 C(W_1(W_1 - \frac{q s}{1 - q} + L^1 D(W_2(W_2 + \frac{q s}{1 - q}) + a[L^1 - 2 L^1 D])]
$$

$$
-L^2 C(W_1(W_1 - s)) + L^1 C(W_2(W_2 + s))
$$

$$
-L^2 C(W_1(W_1 - \frac{q s}{1 - q}) + L^1 C(W_2(W_2 + \frac{q s}{1 - q}) + a[L^1 - 2 L^1 D])]/L^2 (W_2 + s)
$$

As $\partial L_{i-2}^C/\partial s > 0$, and, for $s > (0), L_{i-2}^C > (0), L_{i-1}^D$. A definite pattern for how $E(L_{i-1}^C)$ responds to uncertainty is unclear.

**Proposition 19:** With multiple coverage, under productivity uncertainty and ex-post location flexibility.

**19.1** A unilateral increase in risk will result in a flight of expected labor force out of the inflicted region provided the local demand is not too convex. Expected wage in the economy will rise (decrease) if the demand of the affected region is convex (concave).

**19.2** A generalized and positively correlated across regions increase in risk will result in a flow of labor force out of the region with highest semi-elasticity of labor demand in absolute value, if (but not only if) this increases more rapidly than the other’s. The expected wage will increase (decrease) with risk if convexity (conavity) of the weighted (by wage) sum of the labor demands dominates.

**19.3** A rise in perfectly and negatively correlated labor demand uncertainty will result in a flow difficult to determine. The expected wage will increase (decrease) with risk if convexity (concavity) of the weighted (by wage) sum of the labor demands dominates.

6.2.2. Ex-ante location arrangements

Consider case 2.a: only region $i$ is affected by uncertainty and $Y_j = 0$. For equilibrium:

$$
W_i[q(L(W_1 - s_i) + (1 - q)L'(W_1 + \frac{q s_i}{1 - q_i})], L^1 - L^2_i)
$$

$$
W_iL W_j(L^1 \pm a(L^1 - L^2_i)]L^1_i
$$

Then:

$$
L_{i-1}^C/\partial s_i = q_i W_i[L^1 C(W_1 + \frac{q s_i}{1 - q_i} - L^1 C(W_1 - s_i)], L^1 - L^2_i)/L^1 (W_1 + \frac{q s_i}{1 - q_i})
$$

$$
+ L^2 W_j(L^2 - L^2_i])
$$

Employment will flow to (out of) a region where uncertainty rises if the local demand of the latter is convex (concave).

Consider case 2.b: both sectors are affected by positively correlated uncertainty. For equilibrium:

$$
W_j[q L^1 (W_1 - s) + (1 - q)L'(W_1 + \frac{q s}{1 - q})], L^1 - L^2_i)
$$

$$
W_j[q L^2 (W_2 - s) + (1 - q)L'(W_2 + \frac{q s}{1 - q})], L^1 - L^2_i)
$$

Then:

$$
L_{i-1}^C/\partial s = q[W_i(L^1 (W_1 + \frac{q s}{1 - q} - L^1 (W_1 - s))]
$$

$$
-W_j L_i - L^2_i])
$$

$$
W_i[q L^1 (W_1 - s) + (1 - q)L'(W_1 + \frac{q s}{1 - q})], L^1 - L^2_i)
$$

$$
W_j[q L^2 (W_2 - s) + (1 - q)L'(W_2 + \frac{q s}{1 - q})], L^1 - L^2_i)
$$

Then:

$$
L_{i-1}^C/\partial s > 0 \text{ for } s > 0 \text{ if } W_i(L^1 (W_1 - L^2_i)]L^1 (W_1 + \frac{q s}{1 - q} - L^1 (W_1 - s))] + q(L^1 (W_1 - s) + (1 - q)L^1 (W_1 + \frac{q s}{1 - q} - L^1 (W_1 - s)) - L^2 (W_2 - s)) - L^2 (W_2 - s))]
$$

$$
L^2 (W_2 - s) + (1 - q)L^2 (W_2 + \frac{q s}{1 - q}) + a(L^1 - L^2_i)
$$

$$
\partial L_{i-1}^C/\partial s < 0 \text{ for } s < 0 \text{ if } W_i(L^1 (W_1 - L^2_i)]L^1 (W_1 + \frac{q s}{1 - q} - L^1 (W_1 - s))] = q(L^1 (W_1 - s) + (1 - q)L^1 (W_1 + \frac{q s}{1 - q} - L^1 (W_1 - s)) - L^2 (W_2 - s)) - L^2 (W_2 - s))]
$$

That occurs approximately for $L^1 (W_1)' + L^1 (W_1)' > L^2 (W_2)' + L^2 (W_2)'$. Then, the labor force will flow to the region of more relatively (to its aggregate size) convex labor demand.

Finally, let case 2.c: both regions are affected by negatively correlated uncertainty. For equilibrium:

$$
W_i[q L^1 (W_1 - s) + (1 - q)L'(W_1 + \frac{q s}{1 - q})], L^1 - L^2_i)
$$

$$
W_j[q L^2 (W_2 - s) + (1 - q)L'(W_2 - \frac{q s}{1 - q})], L^1 - L^2_i)
$$
Then:

\[ \frac{\partial L_{\hat{\gamma}}}{\partial s} = q \{ W_1(L_{\hat{\gamma}} - L_{\hat{\gamma}}) [L^1(W_1 + \frac{qs}{1-q}) L^1(W_1 - s)] + W_2 L_{\hat{\gamma}} \{ L^2(W_1 - \frac{qs}{1-q}) - L^2(W_1 - W_2 + s) \} \} \]

\[ + W_2 \{ L^2(W_1 - s) + (1-q) L^2(W_1 + \frac{qs}{1-q}) \} \]

\[ + W_2 \{ q L^2(W_2 + s) + (1-q) L^2(W_2 - \frac{qs}{1-q}) \} + a[L_{\hat{\gamma}} - 2 L_{\hat{\gamma}}] \]  

(79)

\[ \frac{\partial L_{\hat{\gamma}}}{\partial s} > 0 \text{ for } s > 0 \text{ if } W_1 L_{\hat{\gamma}} \{ L^1(W_1 + \frac{qs}{1-q}) - L^1(W_1 - s) \} > W_2 L_{\hat{\gamma}} \{ L^2(W_2 + s) - L^2(W_2 - \frac{qs}{1-q}) \}. \]

Around \( a = 0 \), that is equivalent to \( L^1(W_1 + \frac{qs}{1-q}) - L^1(W_1 - s) \) \( \text{vs} \) \( L^1(W_1 - s) \). Then, the labor force will flow to the region of more relatively (to its aggregate size) convex labor demand.

**Proposition 20:** With multiple coverage and imperfect mobility, productivity uncertainty with ex-ante location decisions:

20.1. A unilateral increase in risk will result in a flight of the labor force to (out of) the inflicted region provided the local demand is convex (concave). In that case, the expected wage in the economy will rise (decrease).

20.2. A generalized and perfectly correlated across regions increase in risk will result in a flow of labor force to the region of more relatively (to its aggregate size) convex labor demand. The expected wage will increase (decrease) with risk if convexity (concavity) of the weighted (by wage) sum of the labor demands dominates.

7. Summary and conclusions

Effects of increasing local risk on migration and labor market equilibrium were inspected in stylized dualistic structures under different mobility assumptions.

Summarizing:

1. A rise in local uncertainty has rarely the same effects as a decrease in a exogenous additive differential accruing to the people living in the area. Such coincidence was only found in totally institutionalized economies or equivalent, and in ex-post decision contexts. That is, an increase in local risk does not necessarily repel the expected local labor force in the long-run – even if a negative shock does in the short-run under flexible labor force responses.

2. In general, a rise in local uncertainty enhances the region’s potential when local demand is convex: that would be compatible with the common knowledge that an increase in risk around the argument of a function increases (decreases) its expected value when the function is convex (concave), that here would apply to demand for aggregate local employment.

As two structures are equilibrating and under different mobility assumptions, that general induction was confirmed in what concerns effects on the average wage in the economy, but excepted in many different cases, made relative or conditional in many different ways, when applied to the explanation of resulting labor flows. Some interesting corollaries on the relation between first derivatives of elasticities, ratios and products of marginal values, and functions’ convexity were remarked along the text.

3. With respect to the impact on labor flows, under free mobility, its direction was determined by the relative size of the (symmetric of the) Arrow-Pratt measure of risk aversion of the labor demands - semi-elasticities of labor demand slopes – of the two regions only under quantity uncertainty and ex-post decision adjustability. In other contexts, the relative size of plain convexity – either of the demands or the inverse demands – would be qualifying.

Of particular interest, the Harris – Todaro model – for which an institutionalized sector coexists with a competitive one - exhibited outcomes dependent on the convexity of functions of the wage bill, or similar conditions, with respect to the (some . . .) argument.

Complete coverage – in the B. H. structure – allowed for the importance of the relative size of plain total employment and of the wage level itself to be determinant – but these conditionings had unique representation - with quantity uncertainty and ex-post adjustability. Under productivity uncertainty, multiple risks with ex-post flexibility forwarded the importance of semi-elasticities of labor demand.

As a final comment we point to the fact that the empirical literature usually does not stress in any way the convexity properties of labor demand functions. In fact, a constant wage-elasticity – of a negatively sloped labor demand - will imply a convex demand, but that is not a requirement deducted by producer’s theory nor a property of all estimable functional forms. Inquiry into the subject would have here realistic applications.

References


