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Ciência & Tecnologia dos Materiais 29 (2017) e254–e261

http://ees.elsevier.com/ctmat

Special Issue "Materiais 2015"

# Theoretical and experimental study of specimens with stress concentrators in dependence of stress triaxiality

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**Abstract**

The literature presents several results referring to multiaxial tests of stress concentrators specimens. Different types of these specimens are presented and they were used accordingly to the stress triaxiality as follows: butterfly specimen for high degree of stress triaxiality, plane specimen for medium state of triaxiality and cylindrical specimen for low degree of triaxiality. This article shows the results of experimental determinations, finite element analysis and theoretical study of two types of plate specimens with stress concentrators and it aims to obtain high stress state in failure section (volume), more uniform stress distribution and stress ratio to be constant until failure.



*Keywords:* Triaxiality degree; finite element analysis; specimens with stress concentrators.

## 1. Introduction

Increased demands concerning performance of materials and structures stimulated the study of materials behaviour, failure criteria and boundary problems that appear in different engineering fields. Given the fact that most of the components have some sort of stress concentrations, several authors have analysed these types of specimens with stress concentrators and developed different limit stress state theories in order to support the technological advances. Basically, stress concentrators can be characterized as a type of discontinuance into the material which alters the stress magnitude. The specimens with stress concentrators are used more often in mechanical studies to underline the importance of stress concentration on an element and the failure mode of the structure. Most of the studies that have been already made [2,3] create a connection between Lode angle, failure envelope, stress triaxiality degree and third invariant.

### 1.1. Stress triaxiality degree

The relation between hydrostatic stress and the equivalent von Mises stress has been defined as a relatively new notion called stress triaxiality:

$$\eta = \frac{\dfrac{\sigma_1 + \sigma_2 + \sigma_3}{3}}{\dfrac{\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}}{\sqrt{2}}} \quad (1)$$

where $\sigma_1$, $\sigma_2$ and $\sigma_3$ are maximum principal stresses.

The degree of stress triaxiality is divided into three categories: low, medium and high stress triaxiality. For each of these categories, the literature has results that show that different types of specimens are correlated to those three groups. Therefore, we are

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http://dx.doi.org/10.1016/j.ctmat.2016.09.001

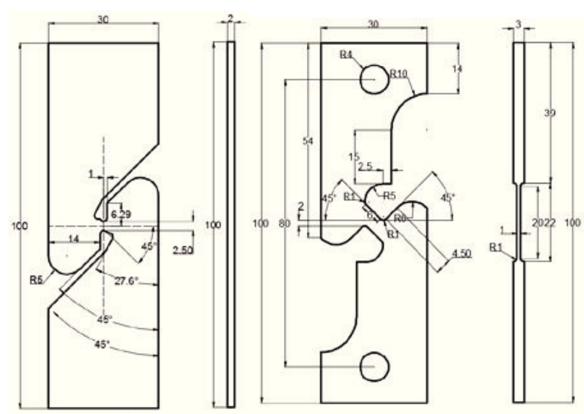
entitled to say that circular specimens with stress concentrators develop high stress triaxiality, plate specimens with stress concentrators develop medium stress triaxiality and butterfly specimens describe low stress triaxiality [1]. The failure of the specimens goes from shearing at low degree of triaxiality until necking at high stress triaxiality. Most of the results for metals have been collected by using biaxial and triaxial tension. The information given by the stress triaxiality degree can predict failure in metallic materials that is caused by plastic deformation.

*1.2. Specimens with stress concentrators*

A stress concentration can be defined as an increment of stress magnitude located in the nearby area of the discontinuity. The stress concentrators are measured by different factors such as local yielding, gradients of stress and strain, beginning of failure state and its development. The stress concentrator's outcome is to modify the distribution of stress, and so we will have high stress values in the vicinity of the discontinuity, but will also describe a decrease of stress with increasing distance from the stress concentrations.
These types of specimens were used in many studies developed by several authors in order to analyse and describe the state of stress for different types of loadings. The results are either from experiments or analytical point of view, but many times they modify the failure criterions due to the importance of that work.
The importance of the shape of the specimen that undergoes buckling was studied by Lamashevskii [4], that concluded that this type of failure depends on the compressive forces, but also on the geometry of the specimen. Further along, Erice [5] made tests under a variation of temperature and used specimens with symmetrical stress concentrators whose triaxiality values were calculated with Bridgman's formula. He used the results to correlate the fracture criterion with fracture models. In addition, by analysing the plastic portion of the true stress–strain curve, the large strain region can be covered by using power-law functions [7]. Conducting the experiments and measuring the equivalent plastic strains to fracture help to calibrate material constants in the ductile fracture criterion for specimens under shear and plane strain tension [8]. However, hydrostatic pressure influences the results [9], which can also be seen in the stress triaxiality degree's equation. Another important factor in the elements failure mode is the growth of voids which will influence the material's stress carrying capacity at large [2]. In order to verify their ability to predict ductile failure under a wide range of stress triaxiality several damage constitutive models have been used (Bai and Wierzbicki, Lemaitre and GTN) [3]. A particular importance is assigned to force versus displacement curves and equivalent plastic strain. Given the advances in the study of materials, the mathematical models become more complex and more precise also.

*1.3. Geometric parameters of the specimens*

The paper presents two types of specimens recreated with different dimensions and materials. Both specimens are plate with asymmetrical stress concentrators, because that will decrease the endowment of the failure. The tests were conducted by biaxial tensile stress.
The first specimen is the one developed by Erice [5], with length of 100 mm, 30 mm wide and 2 mm thickness (Fig. 1 a)) and the second specimen is Bao [6] (Fig. 1 b)) that has a gauge section on the middle of the specimen, in opposition to Erice that has constant thickness. The specimens that finally created used the shape and principles used by the initial researchers but were optimized to meet this study's conditions.




a) b)

Fig. 1. Geometric shape and dimensions.

These types of specimens were chosen after a thorough investigation of literature taking into consideration the shape of the specimen, the material, the stress concentrators and the type of loadings. Since the goals of the paper are to have results for medium stress triaxiality, the specimen's shape had to be plate (Fig. 1).

## 2. Finite Element Analysis

The simulation was made for tensile loading under a uniform loading on one side of the specimen since the other one was anchored. The specimen's boundary conditions were no translation and no rotation on one side and it has a guideline that makes the specimen under simulation to avoid movement and to describe the real testing conditions on a biaxial testing machine. For the finite element analysis, it was used the ALGOR program and the simulation type was "MES with Nonlinear Material Models". The material defined for the finite element analysis was AlMg3 or 5754 Al alloy that has strain hardening modulus of 1560 MPa, yield strength of 289.9 MPa and Young modulus of 67000 MPa. The mesh was of elements type "brick". In the area of stress concentrators, the mesh was made considering a higher number of elements and therefore they have very small dimensions. The first analysis was made in plastic domain using the default generated mesh to examine the failure locus. After that, a new mesh has been made using a symmetrical method that allowed to have some key nodal points in the failure area where we could register the values of stress and strain under loading.

### *2.1. Erice specimen*

Erice specimen was tested in tensile tension, with the force uniformly distributed on the right side (Fig. 2) with a value of 900 N, which has developed a tension of 15 N/mm$^2$. The mesh for this specimen is made of 9355 nodal points and 6960 elements type "brick" and material model "von Mises with Isotropic Hardening".

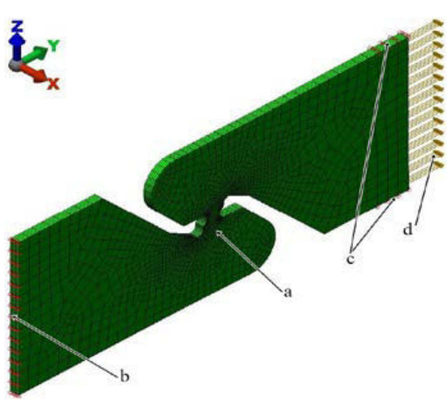



Fig. 2. Loadings and boundary conditions for Erice specimen (a - specimen with stress concentrators, b - embedding, c - guidelines, d - uniform loading).

The distribution of mesh (Fig. 2) and the smaller elements in stress concentrator's vicinity allow an accurate finite element analysis and the possibility to calculate the stress triaxiality degree for the specimen. The results of the finite element analysis are illustrated in the following figures: maximum principal stress (Fig. 3), von Mises stress (Fig. 4) and displacement magnitude (Fig. 5).

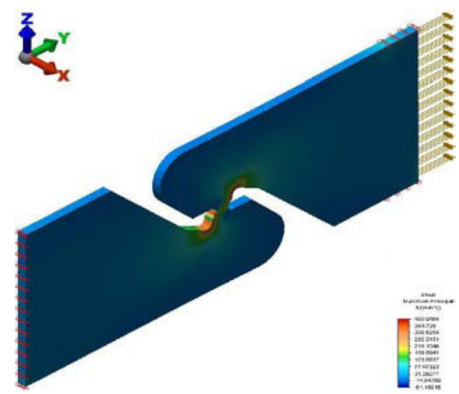



Fig. 3. Maximum principal stress for Erice specimen.

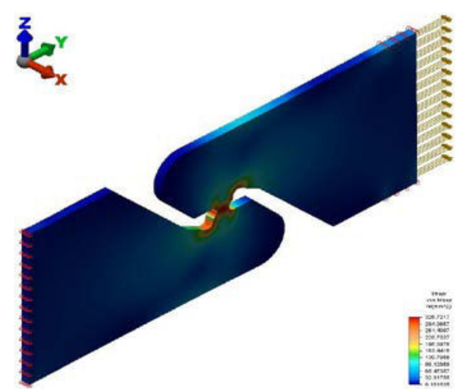



Fig. 4. von Mises stress for Erice specimen.

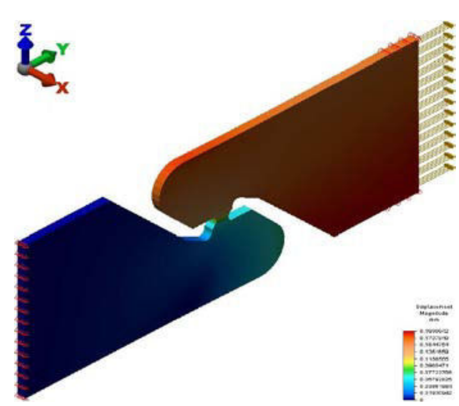



Fig. 5. Displacement magnitude for Erice specimen.

Even though the values of maximum principal stress (400.94 MPa) and von Mises stress (326.72 MPa) are equally important, the failure of the specimen will take place following the path of the shear stress (Fig. 6). In the stress concentrators' area, where in Y-Z critical section achieves a value of 187.99 MPa, they exceed in numerical value and density the other types of stresses.

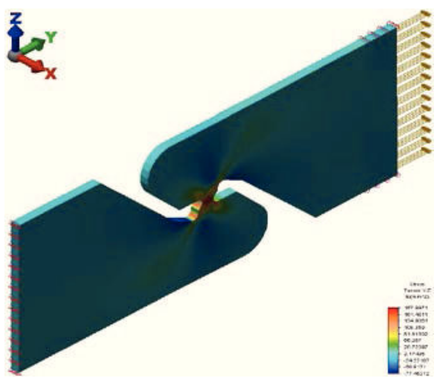



Fig. 6. Shear stress for Erice specimen.

The distribution of shear stress along the specimen's thickness and stress concentrator area (Fig. 7) defines the failure section, which also coincides with the maximum principal stress direction seen in Fig. 8.

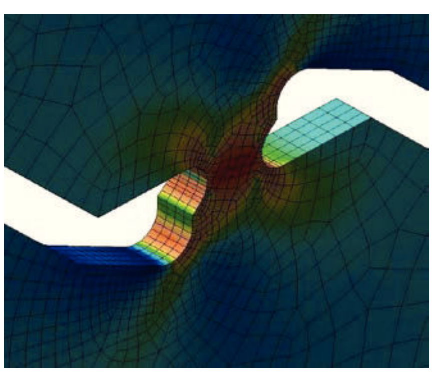


Fig. 7. Detail of the area with stress concentrators when the Erice specimen undergoes shear stress.

When the mesh was remodelled it was chosen a direction at a 45 degree angle in relation to the horizontal axis of the specimen, section considered critic for the stress state. For this particular case, the fracture takes place as a result of shear stress combined with maximum principal stress and von Mises stress and therefore the direction of those stresses was also taken into account.

The yield locus will occur perpendicular on the maximum principal stress directions (Fig. 8) because the material is ductile; otherwise, if the material was brittle, then the fracture would have started on the smallest section of the specimen.

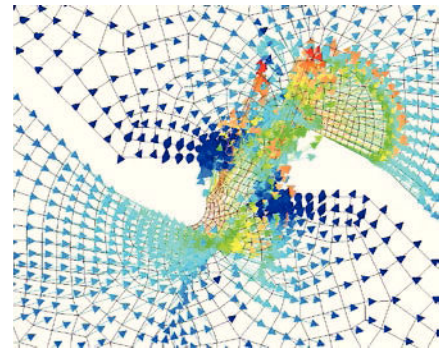


Fig. 8. Maximum principal stress directions (Erice) in 2D.

## *2.2. Bao specimen*

Bao specimen was also tested in tensile tension in the elastic-plastic domain, with the force uniformly distributed on the right side (Fig. 9) with a value of 810 N, which has developed a tension of 9 N/mm$^2$. The mesh for this specimen is made of 9293 nodal points and 6644 elements.

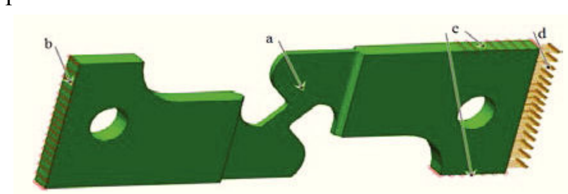



Fig. 9. Loadings and boundary conditions for Bao specimen (a - specimen with stress concentrators, b - embedding, c - guidelines, d - uniform loading).

As for the Erice specimen, the simulation was made to see the results for: maximum principal stress (Fig. 10), von Mises stress (Fig. 11), shear stress (Fig. 12) and displacement magnitude (Fig. 13).

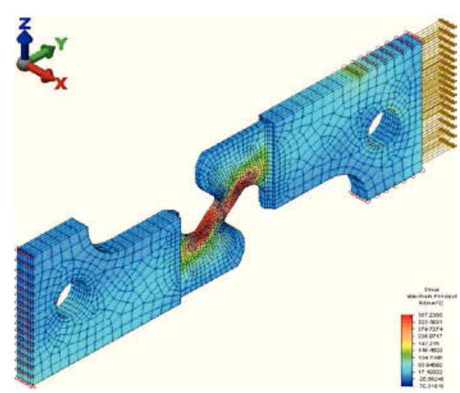



Fig. 10. Maximum principal stress for Bao specimen.

The maximum principal stress values and those for von Mises stress were used to calculate the stress triaxiality degree.

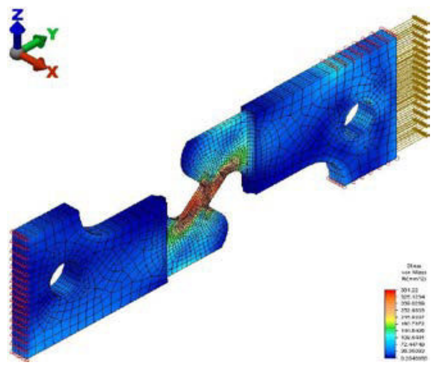



Fig. 11. Von Mises stress for Bao specimen.

The conditions initially imposed on boundary and loadings are observed from the shape of the deformed specimen in such way that it is precise with the real deformation under the testing machine.

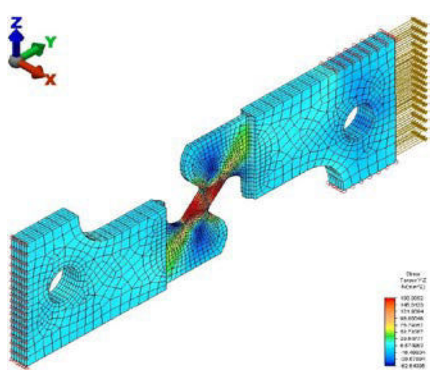



Fig. 12. Shear stress for Bao specimen.

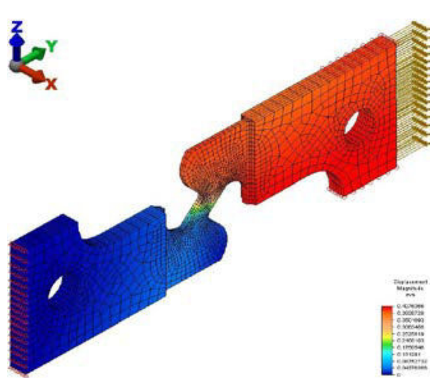



Fig. 13. Displacement magnitude for Bao specimen.

Following the finite element analysis, it is also verified the hypothesis assigned to stress concentrators by which the stress has higher values in the surrounding area of the concentrators than the remaining volume of the specimen. The value of maximum principal stress (367.23 MPa) and von Mises stress (361.22 MPa) are high, but the failure is due to shear stress, which in the stress concentrators' area, in Y-Z plane, achieves a value of 168.08 MPa.
The failure of the specimen takes place due to higher shear stress values in the stress concentrators area (Fig. 14). In order to provide accurate results of the stresses and strains that occur in the specimen under tensile loading, the number of elements is highly increased in the area with stress concentrators.

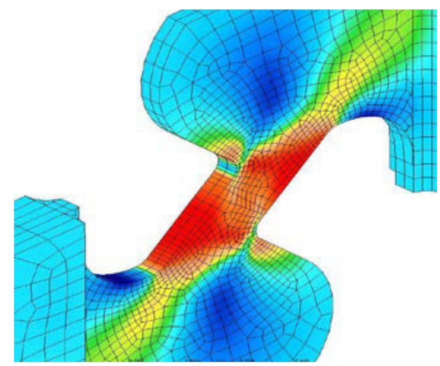


Fig. 14. Detail of the area with stress concentrators when the Bao specimen undergoes shear stress.

Yielding of the specimen happens in this case along the minimum principal stress directions, which are perpendicular to maximum principal stress directions (Fig. 15) and coincide with the critical section at 45 degree that has been created with the help of mesh. The 45-degree critical section considered is taken relative to the horizontal axis of the specimen.

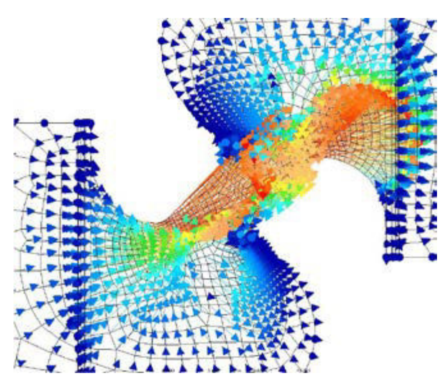


Fig. 15. Maximum principal stress directions (Bao).

## *2.3. Results of finite element analysis*

Following the finite element analysis, the state of stress, strain and displacements was calculated in all nodal points of the specimen. The following diagrams show the variations of: maximum principal stress, von Mises stress, shear stress (Fig. 16, Fig. 17) and displacements (Fig. 18) that occur after the loadings for each type of specimen. The horizontal axis of the diagrams (named "Distance") refers to the 45 degree critical section where the stresses have been registered with decisive values.

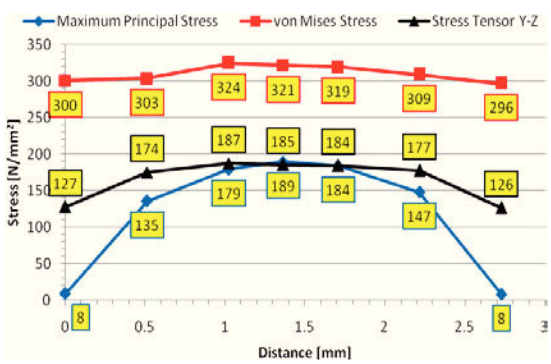



Fig. 16. Stress diagram for Erice specimen.

An important characteristic of the diagram is the rather uniform von Mises stress distribution as a result of the stress concentrator's form in the central section of the specimen.
The area taken into consideration for calculating the stresses was the cross section of the specimen in the above mentioned 45 degree direction.

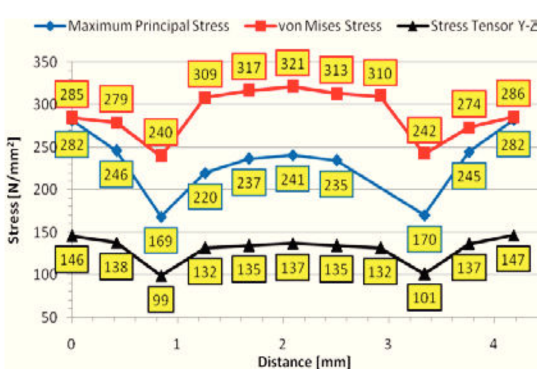



Fig. 17. Stress diagram for Bao specimen.

For 15 MPa, the maximum principal stresses are 26.6 times higher, but for 9 MPa the same stresses are only 40.8 times higher, having a range of similar values. These values fluctuate from 400.90 MPa for Erice specimen to 367.26 MPa for Bao specimen as the high end values and from 189.25 MPa to 241.22 MPa in the middle of the specimen where the failure takes place, respectively. These ratios are relatively high considering the ductile material.

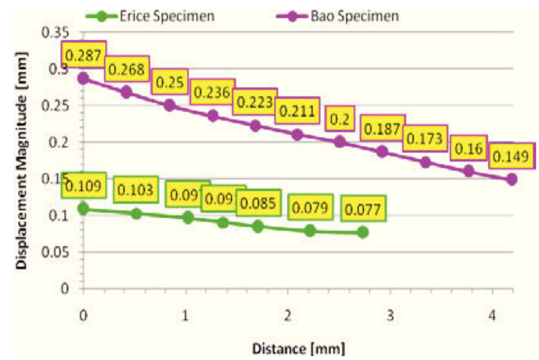



Fig. 18. Displacement magnitude diagram for both specimens.

The variations of the stresses in the specimens under loadings were recorded on a specific direction chosen when the mesh was remodelled so that shear stress has maximum values that will initiate the yielding for ductile materials.

## 3. Experimental Determinations

The experimental determinations took place into three locations: "Gheorghe Asachi" Technical University of Iasi - Faculty of Mechanical Engineering (Strength of Materials Department), Romania, Institute of Macromolecular Chemistry "Petru Poni" of Iasi, Romania, and Instituto Superior Técnico, Departamento de Engenharia Mecânica (DEM), Lisbon, Portugal.
Tests on 10 plate specimens made from AlMg3 alloy - called 5754A, with the following chemical composition: Al 96.5, Mg 3.1, Mn 0.25 and Cr 0.15 (wt.%), were conducted.
The specimens were manufactured through milling process in two batches during the time that experiments took place (Fig. 19 and Fig. 21).

Fig. 19. First batch of specimens: three Erice and one Bao.

First test has been developed on a Erice specimen at Institute of Macromolecular Chemistry "Petru Poni" of Iasi using a 5 kN testing machine with a loading speed of 5 mm/min that is specific for polymers. Next three tests conducted on the remaining specimens from first batch (two Erice and one Bao) took place in the IST laboratory. For these specimens the loading speed was also 5 mm/min under tensile loading (Fig. 20).

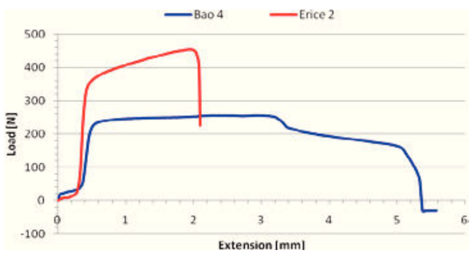



Fig. 20. Load-elongation diagram (specimens 2 and 4).

It is necessary to emphasize the complex character of the loading given by a combination of shear and tension due to the geometry of the specimens (use of asymmetrical stress concentrators).
Having the results from first batch, it was decided that the experiments should continue on some other specimens (Fig. 21) using the same testing machine (Fig. 22) to have same loading conditions, but a different, more adequate, loading speed for metals.

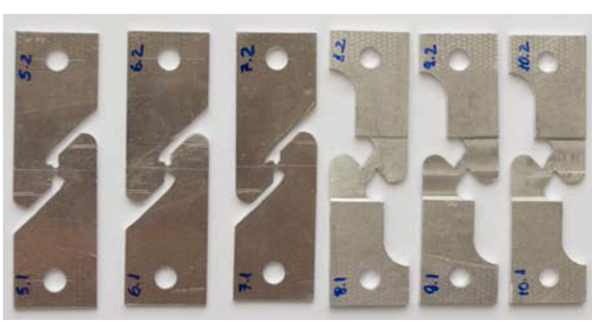


Fig. 21. Second batch of specimens (three Erice and three Bao).

Before the experiments took place, the specimens undergo a microscopical examination in order to look for possible flaws in material, voids or discontinuities. After the mechanical testing, the inspection of the fracture surface of the specimens, to evaluate the quality of material, allowed to conclude that it is a standard material with no impurities, inclusions or visible defects.

Fig. 22. Loading conditions at INSTRON testing machine.

All these last six specimens were tested at a loading speed of 1 mm/min (Fig. 23), which is specific for metals. The testing machine was an INSTRON 5966 of 10 kN capacity.

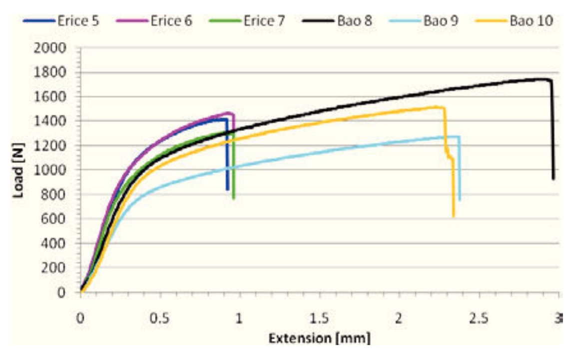



Fig. 23. Load-elongation diagram (specimens 5 to 10).

All specimens have been tested, but the conditions were modified to see if there are differences between results when the machine or the loading speed are changed.
The following diagram (Fig. 24) is a representation of the stress triaxiality degree that is theoretically developed by the Erice and Bao specimens. The calculus was made using the values from the finite element analysis and it shows how, during loading, the specimens develop a medium stress triaxiality that fluctuates into a low triaxiality degree due to the usage and shape of stress concentrators.
After the mathematical calculation of the stress triaxiality factor η, it was indeed determined a mean state of stress taking into consideration hydrostatic stress and the equivalent von Mises stress of the plane specimens. However, the average value of the stress triaxiality degree is 0.118 for the Bao specimen and 0.08 for the Erice specimen.

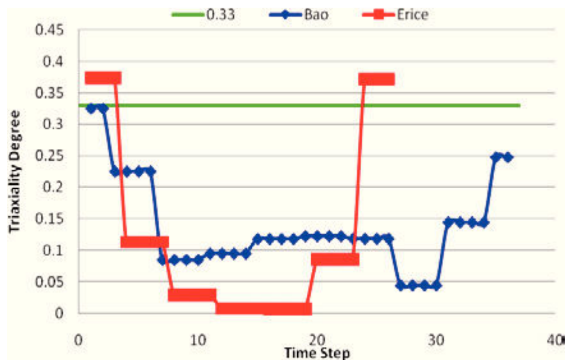



Fig. 24. Stress triaxiality degree development into the two types of specimens.

In literature, the 0.33 value of the parameter is known as the characteristic value in case of medium stress triaxiality (plate specimens) and after the completed experiments at a 1 mm/min loading speed, is noticeable that the results are around that range. Also, each time step presented in Fig. 24 has a value of 20 seconds.

The yielding did not start on the apparent logical short section of the specimen, as it would have happened in case of brittle materials. However, the region where the failure took place registered high values of von Mises stress (324.04 MPa for Erice specimen and 321.37 MPa for Bao specimen), which is the component for stress triaxiality factor.

## 4. Conclusions

Triaxial state of stress can be analysed using three methods: triaxial testing machines, devices that are assembled on the universal testing machines or specimens with stress concentrators; the later method was applied in this paper.

One of the significant criteria in the finite element analysis simulation was the use of elastic-plastic domain. The state of tension achieved in specimen under tensile loading was multiaxial and the consequence was that the failure develops in the close region of the stress concentrators.

von Mises stress has uniform values in the proximity of stress concentrators that are higher than the rest of the specimen volume. This distribution of stresses is a consequence of the asymmetric stress concentrators, which also is the leading cause of the pattern failure caused by principal stress and shear stress mixture in the critic section.

The plate specimens with stress concentrators that have been studied correspond to a medium stress triaxiality state, characterized by accurate prediction of the deformations that commences the yielding and, furthermore, the failure of the specimen.

## Acknowledgements

This work was supported by the strategic grant POSDRU/159/1.5/S/133652, co-financed by the European Social Fund within the Sectorial Operational Program Human Resources Development 2007 – 2013.

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