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Modeling the Forming of Public Opinion: An approach from Sociophysics

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ABSTRACT

This paper reviews a sociophysics two-state model for opinion forming that has proven heuristic power. The dynamics are driven by repeated small-group discussions; within each group, a local majority rule is applied to update the opinions of agents. Iterating the dynamics leads towards one of two opposite attractors at which every agent shares the same opinion. The successful attractor is a function of the initial support with respect to a certain threshold, the value of which depends on the size distribution of the local update groups. While odd-sized groups yield a threshold at fifty percent, even-sized groups, which allow the inclusion of doubt in the case of an opinion tie, produce a threshold shift toward either one of the two attractors, giving rise to minority opinion spreading. In addition, agents can be heterogeneous in their cognitive nature, obeying different rules to update their opinion. While floater agents are open to changing their mind, contrarians chose to oppose whatever opinion was held by the majority of agents in their vicinity, and inflexibles never change their mind. Contrarians and inflexibles have drastic and counter-intuitive effects on the opinion dynamics. Beyond certain critical proportions, contrarians trigger an upside change of the dynamics, making it threshold-less with only one attractor at precisely 50/50 regardless of the initial conditions. Inflexibles produce the same threshold-less dynamics, except with an asymmetric single attractor that favors a specific opinion, even when they start with very low support. The results are used to shed new and unexpected light on controversial issues such as global warming.

Introduction

The field of sociophysics was established in the 1980s (Galam, Gefen, & Shapir, 1982) and for nearly two decades it remained confined to a small number of researchers within the hostile environment of the physics community (Galam, 2004a). Only recently has it become a main stream of research in physics (Chakrabarti, Chakraborti, & Chatterjee, 2006; Galam, 2008a; D. Stauffer, Moss de Oliveira, de Oliveira, & Sa Martins, 2006). Since then, sociophysics has attracted a great deal of research with an increasing number of papers published each year in the main international physical journals (Castellano, Fortunato, & Loreto, 2009). A recent book has explained the basic of sociophysics (Galam, 2012) and a special 2013 issue of the *Journal of Statistical Mechanics* has been devoted to the field (Borge-Holthoefer, Meloni, Gonçalves, & Moreno, 2012; Fortunato, Macy, & Redner, 2013; Galam, 2013; Mauro Mobilia, 2013; Nyczka & Sznajd-Weron, 2013; Sîrbu, Loreto, Servedio, & Tria, 2013; Dietrich Stauffer, 2012).

Sociophysics covers numerous social sciences topics and addresses a wide range of issues, from social networks, language evolution, population dynamics, and the spread of epidemic diseases to terrorism, voting, and coalition formation. Within the field, the study of opinion dynamics has become a main stream of research (Ausloos & Petroni, 2007; Behera & Schweitzer, 2003; Contucci & Ghirlanda, 2007; Crokidakis & Anteneodo, 2012; Deffuant, Neau, Amblard, & Weisbuch, 2000; Ellero, Fasano, & Sorato, 2013; Fortunato & Castellano, 2007; Galam, Chopard, Masselot, & Droz, 1998; Galam & Moscovici, 1991; Galam, 2002, 2004b; González, Sousa, & Herrmann, 2004; Hegselmann & Krause, 2002; Kulakowski & Nawojczyk, 2008; R. Lambiotte & Ausloos, 2007; Renaud Lambiotte, Saramäki, & Blondel, 2009; A. C. R. Martins, 2008; M. Mobilia & Redner, 2003; Pajot & Galam, 2002; Schneider & Hirtreiter, 2005; Slanina & Lavicka, 2003; Solomon, Weisbuch, de Arcangelis, Jan, & Stauffer, 2000; Sznajd-Weron & Sznajd, 2000; Tessone, Toral, Amengual, Wio, & San Miguel, 2004). Moreover, public opinion is now a feature of central importance in modern societies, which makes the understanding of its underlining mechanisms a major

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challenge (A. C. R. Martins, Pereira, & Vicente, 2009; A. Martins, 2008; Vicente, Martins, & Caticha, 2009). Any progress in research on public opinion could have drastic effects on the way in which the sensitive issues currently facing the world are addressed.

The approach the present study has taken to tackle opinion phenomena relies on a few simple assumptions, which in turn provide a series of surprising and powerful results (Galam et al., 1998; Galam, 2002, 2004b, 2005a). In particular, the findings of the present study reveal that the dynamic of opinion formation follows a certain flow, the direction of which appears to be determined by the existence of thresholds in the initial public support for the competing issues. Most models yield such threshold dynamics. Indeed, it has been shown that these dynamics all belong to a single probabilistic sequential scheme (Galam, 2005b).

In 2005, for the first time, a highly improbable political vote outcome was predicted using a sociophysics model (Galam, 2002, 2005a). Moreover, the prediction was made several months ahead of the actual vote and contradicted all polls and analyses predictions (Galam, 2005c). The model deals with the dynamics of spreading of a minority opinion in public debates. It applies to a large spectrum of issues including national votes such as that in the recent French election, behavior changes such as smoking versus non-smoking, support or opposition to a military action such as the war in Iraq, rumors such as the French hoax about 9/11 (Galam, 2003), and reform proposals (Galam, 2000).

The model uses two-state variables to study the formation of a public opinion from a public debate. Agents are floaters who discuss in small groups using a one-person-one-argument principle. At each cycle of discussion, the agents update their individual opinions according to a local majority rule. The associated dynamics are driven by repeated local opinion updates, each of which is followed by a reshuffling of the agents. Local ties may occur in even group sizes. They are solved in favor of either opinion according to the common belief of the agents. The resulting opinion formation is found to be a threshold dynamic with a separator $a_{c,r}$ that determines the direction of the opinion flow towards either one of two attractors a_A and a_B, at which opinions A and B, respectively, are the winning majority. The value $a_{c,r}$ depends on the size r of the update groups. When all agents are floaters, the two attractors are single opinionated with $a_A = 1$ and and $a_B = 0$.

For an opinion A at time t with an initial support $a_t > a_{c,r}$, there exists a number n of successive updates that make the flow to reach the A winning attractor with $a_{t+1} < a_{t+2} < \ldots < a_{t+n} \approx a_A$. In the opposite case, $a_t > a_{c,r}$ leads to a decreasing series $a_{t+1} < a_{t+2} < \ldots < a_{t+m} \approx a_B$, where the number m is different from n. Both are integers that can be calculated exactly with small values when $a_t \neq a_{c,r}$ and a sharp divergence at $a_{c,r}$. Ultimately, only one opinion survives the public debate in the full social community if it lasts long enough to allow the repeated n or m updates to be implemented. Otherwise, for a debate that covers l updates, the final balance of public opinion is given by the respective ending supports for each opinion a_{t+l} and $(1 - a_{t+l})$.

For any odd-size update group, $a_{c,r} = 1/2$. However, even sizes allow a tie to occur, which in turn leads the group into a state of doubt. If there is doubt, the group collective belief is evoked to produce a local bias in favor of either opinion. Such bias may shift $a_{c,r}$ anywhere between 0 and 1 depending on the distribution of both the population collective belief and the local updates group sizes. When $a_{c,r} << 1/2$, the associated dynamics give rise to the occurrence of minority opinion spreading.

In addition, focusing on the no-tie case, for which $a_{c,r} = 1/2$, we study the effect of including heterogeneous agents like contrarians (Galam, 2004c) and inflexibles (Galam & Jacobs, 2007) among the floaters. They are found to have drastic effect on the landscape of the opinion flow driven by the public debate.

Contrarians are agents who deliberately oppose the local majority by shifting to the other opinion, regardless of what the majority opinion is (Borghesi & Galam, 2006; de la Lama, López, & Wio, 2005; Galam, 2004c; D. Stauffer & Sá Martins, 2004; Wio, de la Lama, & López, 2006). At very low densities, contrarians create a stable coexistence between a majority and a minority, with $a_A \neq 1$ and $a_B \neq 0$, the threshold being unchanged at 1/2. However, beyond some critical values, contrarians make the dynamics threshold-less (Galam, 2004c). One unique attractor, $a_A = a_B = a_{c,r} = 1/2$, drives the dynamics. Regardless of what the initial conditions are, the public debate brings the collective opinion to exactly 50/50. This surprising mechanism of threshold erasing was used to explain the famous Bush–Gore presidential election in the US in 2000. It was then predicted that 50/50 elections would often occur again in voting democracies. In fact, it has happened on several subsequent occasions, such as in Germany, Italy, and Mexico (Galam, 2007). The majority level at which contrarians operate can be global instead of local using results from polls. It gives rise to chaotic behavior around 50% (Borghesi & Galam, 2006).

Inflexible agents never change their opinions during small-group discussions. They produce similar effects as contrarians, but with the novelty of asymmetry since the densities of inflexibles for each opinion are usually not equal. This is particularly the case considering that one-sided inflexibles make the associated opinion certain to gain the full support of the population. Even if an opinion is supported by only a very low density of inflexibles, as opposed to a huge majority of floaters in favor of the other opinion, the debate will reverse the ratio with the entire population eventually aligned with the inflexibles' opinion.

Accordingly, the expected democratic character of a free public debate may turn into a dictatorial machine to propagate the opinion of a tiny minority against the initial opinion of the overwhelming majority. It may shed new and counter-intuitive light on the social aspect of controversial issues such as global warming (Galam, 2008b).

The remainder of this paper is organized as follows. Section 2 presents the local majority model with only floaters; update groups are all of the same size r. It is found that the $r = 2$ case may exhibit a threshold-less dynamics. The combination of various sizes is then investigated. Section 3 deals with heterogeneity of the agents with the possibility of contrarian behavior. A threshold-less dynamic is obtained with a perfectly balanced coexistence of both opinions. One-sided inflexible agents are introduced in Section 4. Above a small concentration of one-sided inflexibles, the dynamic is threshold-less, which makes the favored opinion certain to gain support from the full population through the public debate. Section 5 contains some discussion about the perspectives with which to make sociophysics a predictive solid field of social science with an emphasis on both the challenges and the difficulties. The results are shown to shed a new light on controversial public issues such as global warming. Extensions and limits of the approach are discussed briefly.

The local majority model

The model consists of a group of N agents undergoing a public debate. Each agent $i = 1, \ldots, N$ holds either one opinion A or B, denoted by $a_i = \pm 1$, respectively. Before the public debate began, each agent made an individual choice according to its public and private information, including its own belief. When the public debate is launched at time t, the initial proportions of both opinions are a_t and $(1 - a_t)$, respectively, and are obtained using polls.

The opinion dynamic is then driven by a series of repeated cycles of local discussions during which agents eventually update their own opinion. At each update, random groups of r agents are formed. Within each group, all agents adopt the opinion that has the local majority. Group size r may vary with $r = 1,2,\ldots L$.

However, local majority rule does not apply in case of a tie in an even-numbered size group. In that case, a common belief "inertia principle" is applied in order to select either one opinion A or B, with respective probabilities k and $(1 - k)$, where k accounts for the collective bias produced by the common belief of the group members.

A k non-integer value accounts for the fact that real societies are divided into disconnected subgroups, which may share different collective beliefs and some members not discussing with all other members (Galam, 2005a). For instance, in case of a reform proposal, most cultures share the norm that, in case of doubt, it is better to retain the status quo, which would put $k = 0$ if opinion A corresponds to "yes" to the reform. For other issues, one social subgroup may have a common belief yielding $k = 1$, while another has an opposite common belief with $k = 0$, resulting on average in an effective bias $0 < k < 1$.

Accordingly, one cycle of local update leads to new proportions a_{t+1} and $(1 - a_{t+1})$, with

$$a_{t+1} = \sum_{m=\frac{r+1}{2}}^{r} r_m \, a_t^m (1 - a_t)^{r-m} \qquad (1)$$

for odd sizes, and

$$a_{t+1} = \sum_{m=\frac{r}{2}+1}^{r} r_m \, a_t^m (1 - a_t)^{r-m} + k a_t^{\frac{r}{2}} (1 - a_t)^{\frac{r}{2}} \qquad (2)$$

for even sizes, where $r_m \equiv \frac{r!}{m!(r-m)!}$ is a binomial coefficient.

This local majority rule mechanism was first introduced in the 1980s to study bottom-up hierarchical voting models (Galam, 1986). In such models, groups of agents designate representatives at a higher hierarchical level using local majority rule and, in case of a tied vote, a status quo inertia is applied to retain the existing representative.

To study the dynamics associated with the repeated updates, we solve the fixed point equation $a_{t+1} = a_t$. The equation yields two attractors, $a_A = 1$ and $a_B = 0$, and a threshold $0 < a_{c,r} < 1$, which separates the flow opinion in direction of either $a_A = 1$ or $a_B = 0$ as shown in Figure 1 depending on the location of a_t with respect to $a_{c,r}$. If $a_t > a_{c,r}$, one update gives $a_{t+1} > a_t$; otherwise, by symmetry, $a_t < a_{c,r}$ yields $a_{t+1} < a_t$. From the first case, repeating the update gets the support closer to the attractor $a_A = 1$. Indeed, there is a number n to reach it with the series $a_t < a_{t+1} < a_{t+2} < \ldots < a_{t+n} = a_{n+1} = a_A = 1$, at which an equilibrium state is obtained with only opinion A within the full population. The other opinion has totally disappeared. The reverse holds true when $a_t < a_{c,r}$.

While for odd size groups $a_{c,r} = 1/2$, for even sizes the possibility of local doubt at a tie breaks the symmetry between opinion A and B according to the value of k, which is determined by the distribution of collective belief in the population. The threshold value a_c depends on r and k (Galam, 2005a). Figure 1 exhibits the three cases with $a_{c,r} = 1/2$ (top), $a_{c,r} < 1/2$ (middle), and $a_{c,r} > 1/2$ (bottom). For odd sizes and for even sizes with $k = 1/2$, we have $a_{c,r} = 1/2$.

Solving the problem exactly in the case of groups of size $r = 4$ yields for the threshold value

$$a_{c,4} = \frac{6k - 1 - 13 - \sqrt{36k+36k^2}}{6(2k-1)} \qquad (3)$$

which yields $a_{c,4} = 0.23, 0.77, 1/2$ for $k = 1, 0, 1/2$, respectively. This value shows explicitly how the existence of doubt combined with a collective belief that favors one opinion (A, in this case), can turn the public debate into a machinery to propagate a minority opinion. When $k = 1$ to win the final majority, the initial A support must satisfy only $a_t = 0.23$.

Increasing the group size r dampens this effect by making $a_{c,r}$ closer asymptotically to the value 1/2. For instance $k = 1$ gives $a_{c,r} = 0, 0.23, 0.35, 0.39, 0.42, 0.44, 0.45, 0.47, 0.46, 0.47$ for $r = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20$, as reported in Figure 2. Nevertheless, it is worth emphasizing that people always discuss in small groups. Larger groups usually split into smaller ones, as readily observed during dinner parties involving more than six persons.

The Singular Feature of Pair Interactions

From Figure 2 it appears that while the threshold value decreases with smaller group size, it reaches zero at $r = 2$, i.e., for pairwise discussions. This implies a rather strong effect in the opinion dynamics bias driven by discussions within couples. Indeed, many exchanges occur in pairs. It is noticeable that for pairwise groups the

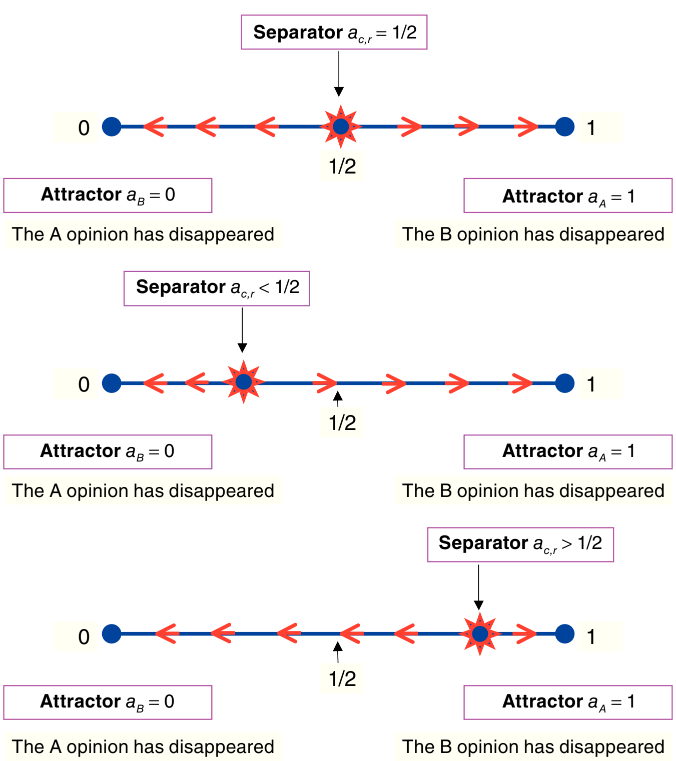

Figure 1. The opinion flow for the A opinion with $a_{c,r} = 1/2$ (top), $a_{c,r} < 1/2$ (bottom left), and $a_{c,r} > 1/2$ (bottom right).

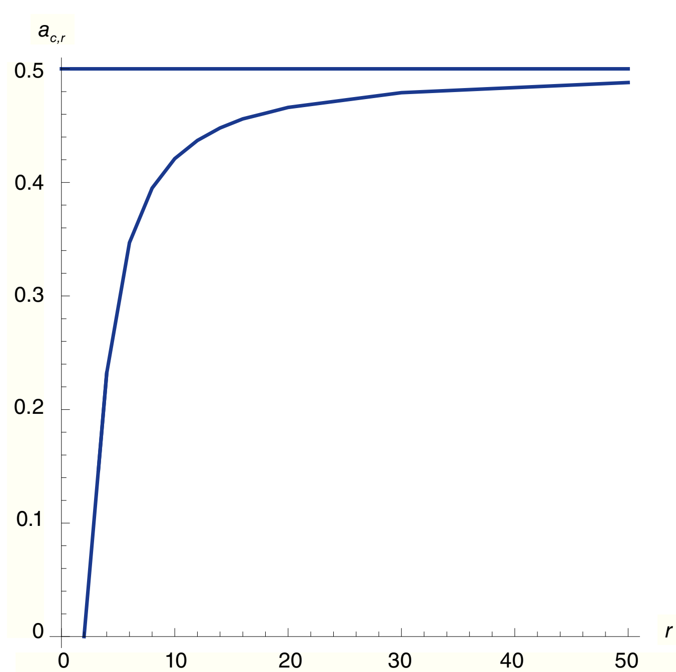

Figure 2. The threshold $a_{c,r}$ as a function of even size $r = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 30, 50$ for $k = 1$.

threshold $a_{c,r} = 0$; in other words, a few agents supporting A opinion is enough to invade the whole population, given $k = 1$. Any initial support for A and B respectively ends up with the entire population sharing opinion A. More precisely, Eq. 2 is written as

$$a_{t+1} = a_t^2 + 2ka_t(1 - a_t). \quad (4)$$

It yields two opposite regimes, both of which are threshold-less, as Figure 3 shows. In the range $0 \leq k < 1/2$, the separator is $a_{c,2} = 1$ and

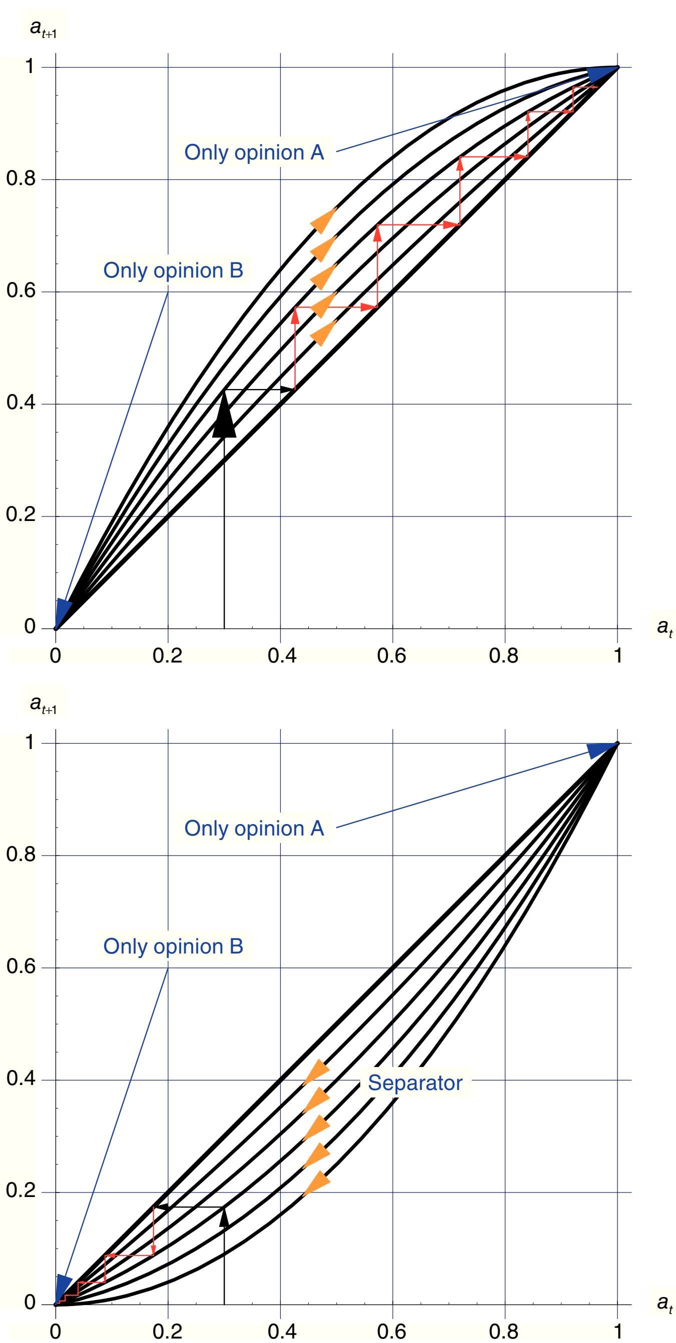

Figure 3. The variation of a_{t+1} as a function of a_t from Eq. (4). The left part includes the series $k = 1$ (above external line), $k = 0.90$ (upper inside), $k = 0.80$, $k = 0.70$, $k = 0.60$, and $k = 1/2$ (the diagonal straight line). Any initial A support $a_t \neq 1$ is expanded through the public debate towards total invasion at $a_A = 1$. The left part includes the series $k = 0$ (below external line), $k = 0.10$ (below inside), $k = 0.20$, $k = 0.30$, $k = 0.40$, and $k = 1/2$ (the diagonal straight line). Any initial A support $a_t \neq 0$ is shrunk through the public debate towards total disappearance at $a_B = 0$. The opposite dynamics associated to an initial A support $a_t = 0.30$ is shown for both cases $k = 0.80$ (upper part) and $k = 0.20$ (below part).

the unique attractor is $a_B = 0$. Any initial condition, apart from $a_t = 0$, leads to $a_{t+n} = 0$. By contrast, for $1/2 < k \leq 1$, we have the separator $a_{c,2} = 1$ with the unique attractor $a_A = 1$. Any initial condition apart from $a_t = 0$ leads to $a_{t+n} = 1$. The case $k = 1/2$ produces an invariant dynamics with $a_{t+1} = a_t$. Agents change their opinions individually, but on average the global supports do not.

It should be stressed that the effect outlined above has been corroborated by an analysis of data from the 2004 American presidential election regarding the marriage gap;that is, the difference in voting for Bush and Kerry between married and unmarried people. It appears that this marriage gap can be interpreted in terms of our model with a positive value of k if Bush is denoted by opinion A (Kulakowski & Nawojczyk, 2008).

Mixing the Group Sizes

In real life, people do not always discuss in successive groups of the same size. Therefore, to make our model more realistic, we consider the distribution given by the probability p_r to have a local group of size r with the constraint

$$\sum_{r=1}^{L} p_r = 1, \quad (5)$$

where $r = 1,2,\ldots,L$ stands for respective sizes $1,\ldots,L$ with L being the size of the larger group (Galam, 2002). Accordingly, the general update Equation is

$$a_{t+1} = \sum_{r=1}^{L} p_r \left\{\sum_{j=N[\frac{r}{2}+1]}^{r} C_j^r a_t^j (1 - a_t)^{r-j} + kV(r) C_{\frac{r}{2}}^r a_t^{\frac{r}{2}} (1 - a_t)^{\frac{r}{2}}\right\} \quad (6)$$

where $C_j^r \equiv \frac{r!}{(r-j)!j!}$, $N[\frac{r}{2}+1] \equiv$ Integer Part of $(\frac{r}{2}+1)$ and $V(r) \equiv N[\frac{r}{2}] - N[\frac{r-1}{2}]$. It gives $V(r) = 1$ for r even and $V(r) = 0$ for r odd.

The occurrence of local ties in even-size groups from Eq. (6) produces an asymmetry in the polarization dynamics towards either one of the two competing opinions, with a threshold $a_{c,\bar{r}}$, which may be very unfair for one of the two opinions. We illustrate the process choosing the set $p_1 = 0.10$, $p_2 = 0.25$, $p_3 = 0.10$, $p_4 = 0.30$, $p_5 = 0.10$, $p_6 = 0.15$ for the size distribution, which yields $a_{c,\bar{r}} = 0.214,1/2,0.786$ for respectively $k = 1,1/2,0$.

Figure 4 illustrates the above set of p_r for the three different cases $k = 1,1/2,0$ with the same initial minority support $a_t = 0.30$. When the collective belief favors opinion A, its initial support $a_t = 0.30$ is seen to increase drastically driven by the public debate. Only four updates are sufficient to make opinion A the majority. For a neutral tie effect — that is, $k = 1/2$ — the value decreases towards zero, while for a bias in favor of opinion B, it falls off very quickly to zero support as Figure 4 shows.

Heterogeneous agents and the contrarian effect

So far, we have considered identical behavior for all agents to make up their opinion. Each agent gives its own opinion in a discussing group, and then all agents eventually shift towards the same opinion; that is, the one that is supported by the majority of arguments. In case of a local tie, it is the collective belief that sets up a bias in favor of one specific choice. However, it is clear that in real-life situations, agents may exhibit different types of behavior in their cognitive process of opinion selection .

One type of individual feature is observed when an agent determines its choice, not with respect to its own view, but simply to oppose what the others are advocating (Borghesi & Galam, 2006; Galam, 2004c). We refer to such agents as contrarians. Whatever the prevailing choice of the majority is , a contrarian adopts the opposite choice. In the case of update groups of odd size, Eq. (1) changes to

$$a_{t+1} = (1 - 2c) \sum_{m=\frac{r+1}{2}}^{r} r_m\, a_t^m (1 - a_t)^{r-m} + c, \quad (7)$$

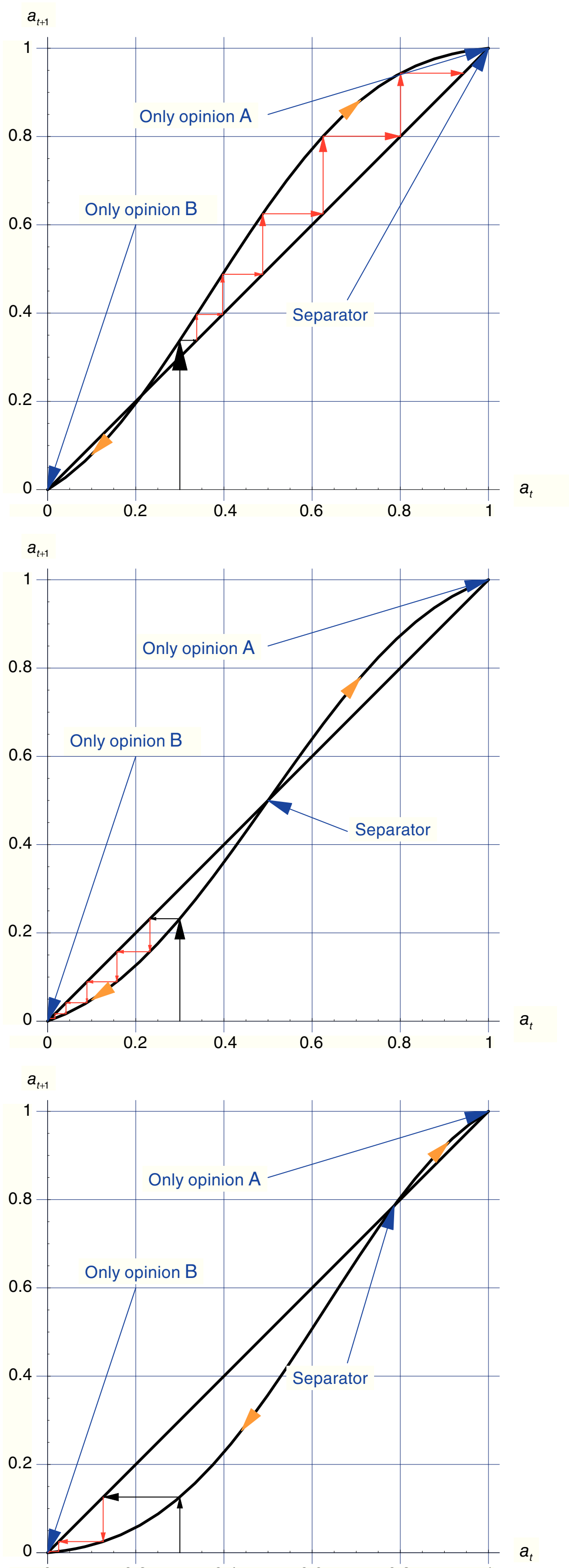

Figure 4. The variation of the general update from Eq. (6) for the set of size distribution $p_1 = 0.10$, $p_2 = 0.25$, $p_3 = 0.10$, $p_4 = 0.30$, $p_5 = 0.10$, $p_6 = 0.15$ and an initial A support $a_t = 0.30$. The top left part corresponds to $k = 1$ which yields $a_{c,r} = 0.214$. The A minority wins the majority rather quickly. The top right part has a neutral tie with $k = 1/2$ which yields $a_{c,r} = 1/2$. The A support decreases towards zero. The bottom part shows the case of the tie breaking in favor of the B opinion with $k = 0$ and $a_{c,r} = 0.786$. The A support disappears fast.

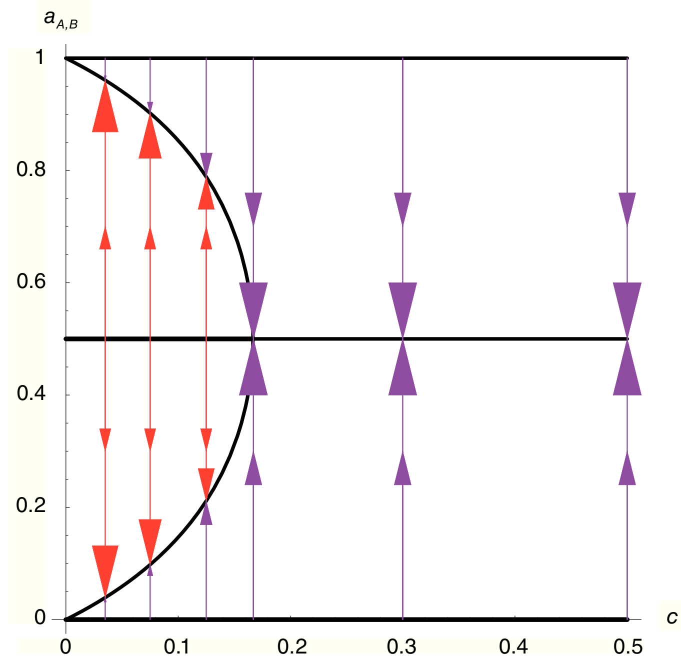

Figure 5. The contrarian two attractors $a^c_{A,B}$ from Eq. (8) as a function of contrarian density c. At $c = 1/6$ both a^c_A and a^c_B merge with the separator $a_{c,3} = 1/2$ to turn the dynamics threshold-less. In the range $1/2 \leq c \leq 1/2$, the dynamics leads systematically to a fifty-fifty coexistence between opinions A and B.

where c is the density of contrarians among the floater population. The associated dynamic is changed since $a_t = 0$ and $a_t = 1$ yield $a_{t+1} = c$ and $a_{t+1} = 1 - c$, respectively. These values are no longer fixed points of the Equation $a_{t+1} = a_t$. However, the status of $a_{c,r} = 1/2$ is unchanged and remains a fixed point.

Solving the fixed point Equation $a_{t+1} = a_t$ for $r = 3$ using Eq. (7) yields

$$a^c_{A,B} = \frac{1-2c \pm \sqrt{1-8c+12c^2}}{2(1-2c)} \tag{8}$$

as depicted in Figure 5. As the figure shows, at low densities of contrarians, increasing c makes both attractors to move symmetrically towards the threshold $a_{c,3} = 1/2$ with $a^c_A < 1$ and $a^c_B > 0$. These are found to all merge with $a^c_A = a^c_B = a^c_{c,3} = 1/2$ at exactly $c = 1/6$. In the region $0 \leq c \leq 1/6$, the opinion dynamic yields a stable coexistence between a majority A (B) and a minority B (A).

Figure 6 exhibits the two cases $c = 0.05 < 1/6$ and $c = 0.35 > 1/6$. The evolution of $a_t = 0.30$ is shown in both cases. The first one leads to a majority B and a minority A, while the second produces 50/50 support.

Threshold-Less Driven Coexistence

From $c = 1/6$ and beyond in the range $1/6 \leq c \leq 1/2$, the dynamic becomes threshold-less. Instead of having a flow that puts one of the two competing opinions ahead, the dynamic reduces any initial difference to zero, establishing a perfect equality between the two competing opinions A and B, as seen in Figure 5. The two attractors a^c_A and a^c_B have disappeared in favor of one unique attractor located at the value $a_{c,3} = 1/2$ of the former separator.

Driven by the local discussions, once the equilibrium is reached, agents continue to update their opinion, due to the contrarians who never settled down. However, the net effect of these individual opinion changes is perfectly self-balanced without shifting the global equality of 50/50.

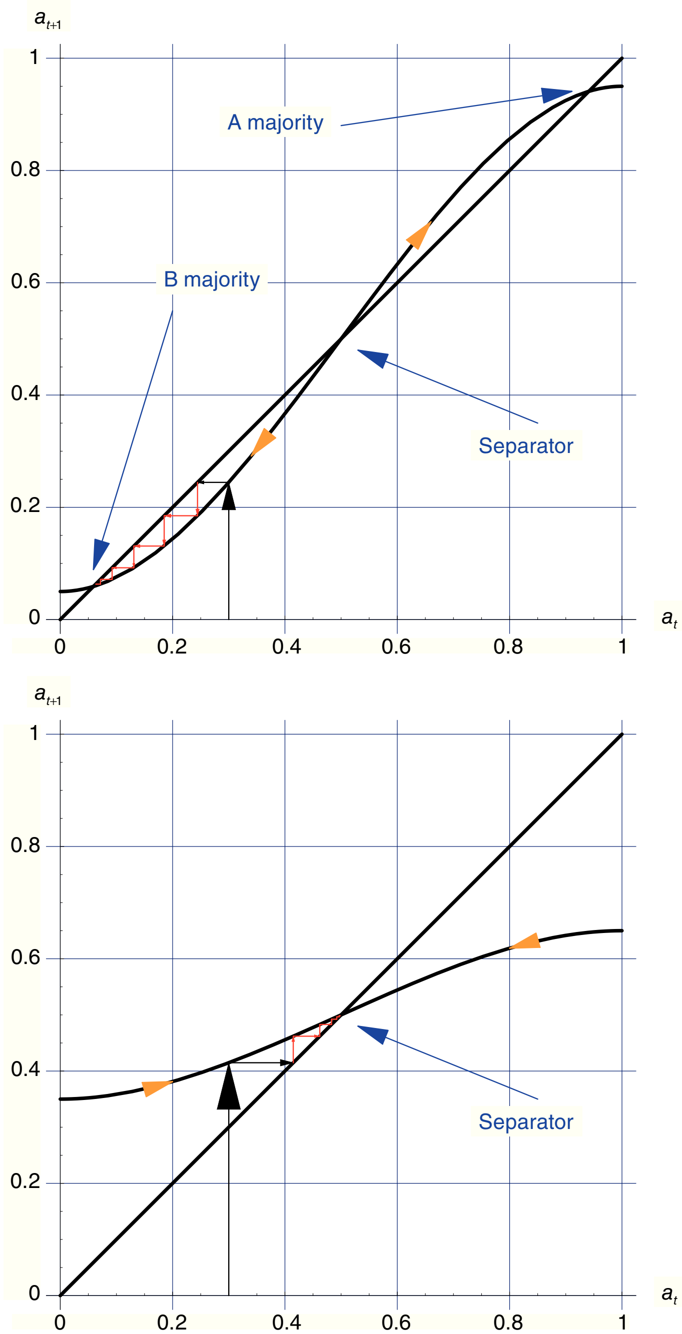

Figure 6. The evolution of a_{t+1} as a function of a_t for $c = 0.05 < 1/6$ (left part) and $c = 0.35 > 1/6$ (right part). The evolution of $a_t = 0.30$ is shown in both cases. The first one leads to a B majority and a A minority while the second one produces a fifty/fifty support.

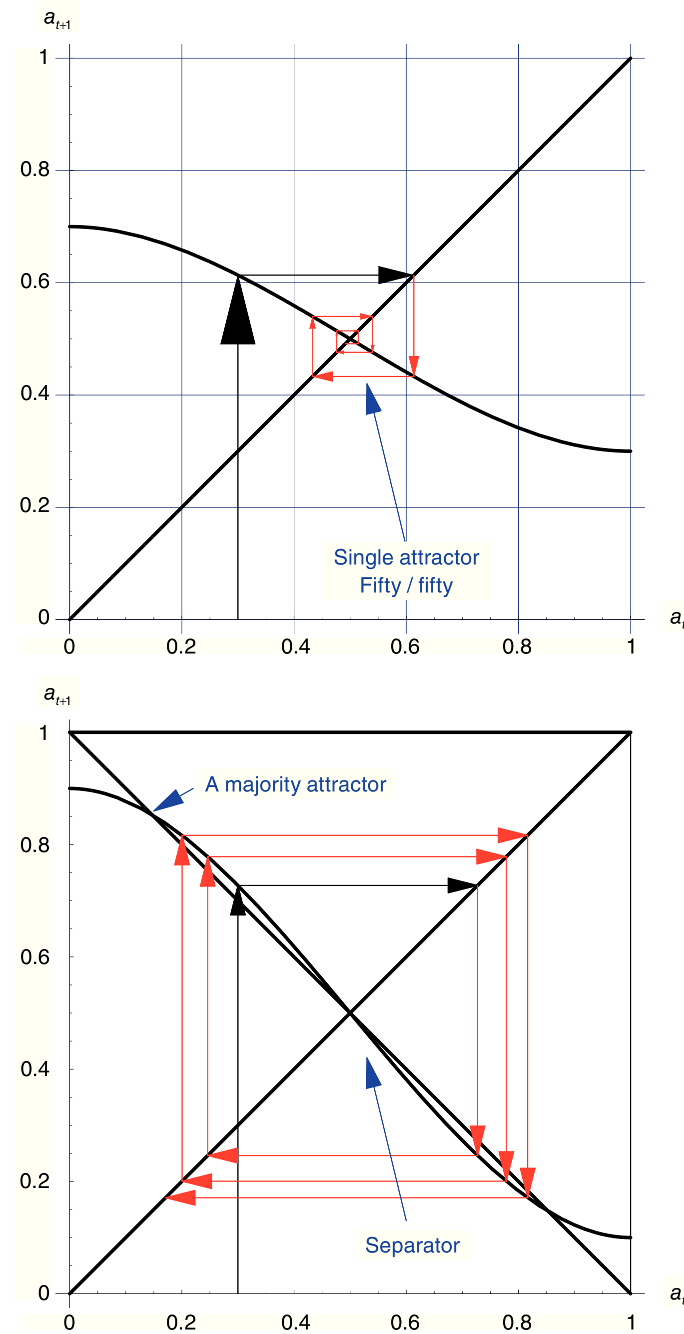

Figure 7. The alternating opinion dynamics for $c > 1/2$. The left part shows the case $c = 0.70$ for which the dynamics is threshold-less with a convergence towards the attractor fifty/fifty. The right part exhibits the case $c = 0.90$ where there exists alternating attractors. The evolution of $a_t = 0.30$ is shown for six successive updates in both cases.

For larger values of the contrarian density with $c > 1/2$, the dynamics start to alternate. In the range $1/2 \leq c \leq 5/6$, the majority shifts from one opinion to the other one at each new update, but the difference in amplitude reduces until it reaches zero at the same attractor 1/2. Beyond this, with $5/6 < c \leq 1$, the dynamic continues to alternate, but now 1/2 is again a separator with two alternating attractors $a_A^c < 1$ and $a_B^c > 0$. Figure 7 exhibits the two cases, $c = 0.70$ and $c = 0.90$, for an initial A support $a_t = 0.30$.

The contrarian behavior has also been extended to consider that the contrarian determines itself, not with respect to the majority within its discussing group, but with respect to the global majority at the collective level given by polls (Borghesi & Galam, 2006). The effect is similar to the previous one, except that a chaotic behavior is now obtained around 50%.

It is worth noting that any real election held in that 50/50 equilibrium state within a large size group will yield an outcome that will never be exactly 50/50 due to incompressible counting error or tiny fraud scoring less than the expected statistical fluctuations. While these small amounts do not usually affect the result, being at 50/50, they will make any victory extremely narrow, and therefore disputable (Galam, 2007).

The one-sided inflexible effect

Another specific feature of human character is the inflexible attitude. An inflexible agent sticks to its opinion regardless of what arguments it is given (Galam & Jacobs, 2007). The motivation can take various forms, such as conviction, lie, interest, or fear. The effect on the dynamics is found to be similar to the contrarian effect, but with the introduction of an asymmetry between the two opinions A and B, depending on the the ratio in the respective proportions of inflexibles. In particular, the separator is no longer located at 1/2 as for contrarians and the two attractors are not symmetric (Galam & Jacobs, 2007).

While it seems natural to have inflexible agents on both sides of any social issue, some peculiar cases do not obey this logic. This particularly applies to issues for which some agents are convinced to have "scientific proof" to justify their opinion only against doubts prevailing on the other side, as is the case with human-caused global warming (Galam, 2008b).

In this example, some agents believe that scientific proof has been obtained to link the global warming to the man-made production of carbon dioxide. On the other hand, a smaller number of agents claim there is no scientific proof of human culpability, although they have neither proof of another cause nor proof that humans are not guilty. In between these two groups, the majority of agents are floaters.

To model the public debate associated to such a situation, we consider a proportion q of inflexible agents in favor of opinion A, with other agents being floaters who obey a local majority rule. We also assume that the majority of floaters initially support opinion B. For $r = 3$, instead of Equation (1), the corresponding update becomes

$$a_{t+1} = a_t^3 + 3a_t^2(1 - a_t) + q(1 - a_t)^2 \quad (9)$$

where last term accounts for the configurations where two opinion B holders discuss with one inflexible A agent, yielding the respective opinions unchanged by the local update. It should be stressed that $a_t \leq q$ by definition of the inflexibles.

To study the effect on the dynamics, we solved the new fixed point Equation $a_{t+1} = a_t$ using Eq. (9), which yields two attractors, $a_A = 1$ and

$$a_B^q = \frac{1}{4}(1 + q - \sqrt{1 - 6q + q^2}), \quad (10)$$

with a separator located at

$$a_{c,3}^q = \frac{1}{4}(1 + q - \sqrt{1 - 6q + q^2}), \quad (11)$$

both a_B^q and $a_{c,3}^q$ being defined only in the range $0 \leq q \leq q_c \equiv$ $\equiv 3 - 2\sqrt{2} \approx 0.172$. At q_c we have $a_B^q = a_{c,3}^q \approx 0.29$.

The first effect of the one-sided inflexibles is to prevent the total disappearance of opinion A. Even in the extreme case with all the floaters supporting opinion B, apart from a tiny proportion $q < q_c$ of A inflexibles, an incompressible A minority with a proportion a_B^q will survive the debate. Due to the random distribution of infexibles, they produce constantly some individual shifts among floaters from B to A, as shown in Figure 8. The increase is given by

$$a_B^q - q = \frac{1}{4}(1 - 3q - \sqrt{1 - 6q + q^2}), \quad (12)$$

which reaches its maximum value ≈0.29 at q_c; that is, a net increase of ≈0.29 − 0.17 = 0.12.

However, in the range $0 \leq q \leq (3 - 2\sqrt{2}) \approx 0.172$, any A support satisfying $a_t < 0.172$ leads the B opinion to eventually win the public

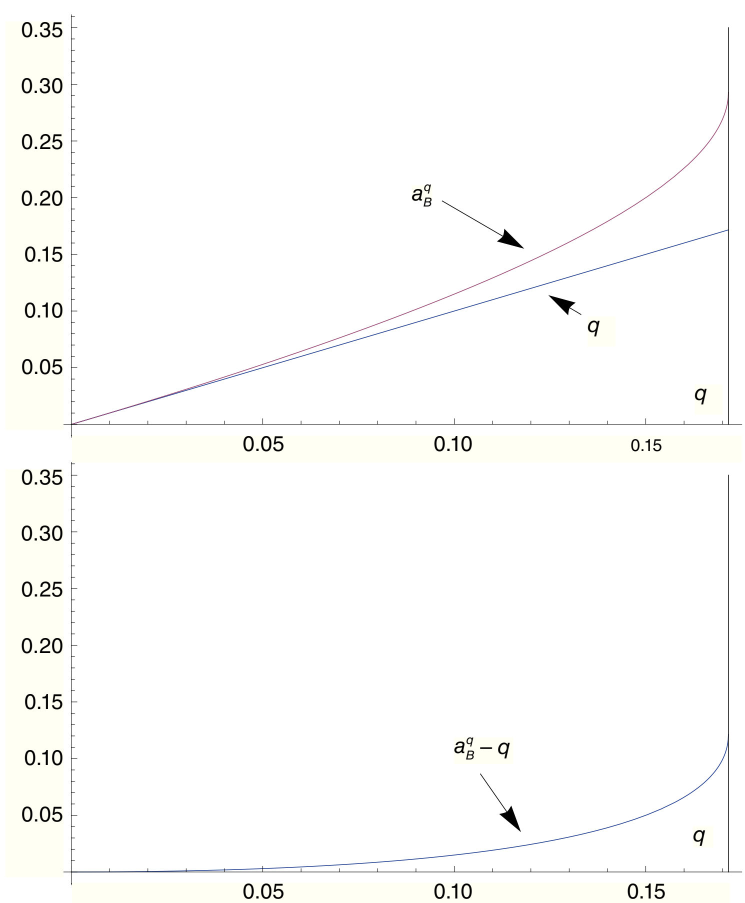

Figure 8. The proportion of A supporters driven by the public debate starting from a tiny proportion q of inflexibles in the range $0 \leq q \leq (3 - 2\sqrt{2}) \approx 0.172$ (left part). The net proportions of floaters gained by the random distribution of inflexibles over the same range (right part).

debate. On the contrary, any initial support that satisfies $a_t > 0.172$ ends ups with a total victory of A opinion, even with 80% of initial support for opinion B. The process is illustrated in the upper part of Figure 9, with $q = 0.05$ and $a_t = 0.30$.

The Threshold-Less Case

Increasing the inflexible density q from zero, the two fixed points a_B^q and $a_{c,3}^q$ move towards one another to eventually merge at $q = q_c$ with $a_B^q = a_{c,3}^q \approx 0.293$ and then disappear for $q > q_c$. In that case, any B support, even if it is huge, systematically loses the public debate against an ultra-small A minority that includes at least a little more than 17% of inflexibles.

One example is exhibited in the bottom part of Figure 9 with the two cases $q = 0.05 < q_c\ (3 - 2\sqrt{2}) \approx 0.172$ (left part) and $q = 0.20 > (3 - 2\sqrt{2}) \approx 0.172$ (right part) for the same initial proportion $a_t = 0.30$ of A support. Using Equation (9) brings down the support in favor of A o 0.053 in the first case, but boosts it to high scores in the second case. Only a difference of 15% of inflexibles in the composition of the 30% percent of A support makes such a huge contrast in the final outcome of the public debate. Adding more updates will bring the A support to 100% in the second case.

Figure 10 shows the various regimes for $q < q_c$ and $q > q_c$ as a function of q. The grey area of the figure represents the incompressible A inflexible domain. The A attractor $a_A = 1$ is independent of q and stayed unchanged, while it also becomes the unique attractor of the dynamics as soon as $q > q_c$.

Conclusion

Sociophysics is a promising field due to its specific capacity to reproduce some complex social situations within a new coherent frame with the discovery of novel and counter-intuitive dynamics, which could be active in the social reality.

With respect to opinion dynamics, sociophysics emphasizes the biases that may be involved in holding a public debate to decide on major social issues. Within our model, we saw how a free public debate naturally produces a dictatorial machine to propagate the opinion of a tiny minority against what could have been the initial opinion of an overwhelming majority. While rationality is applied using a majority rule, the possible occurrence of local doubts opens the way for the collective belief to make the choice. In case of a threshold dynamic, these collective belief biases can shift the threshold from 50% to any side at values ranging from 10% to 90%.

The model applies to a large range of social, economic, and political phenomena, particularly effects like propagating fear and

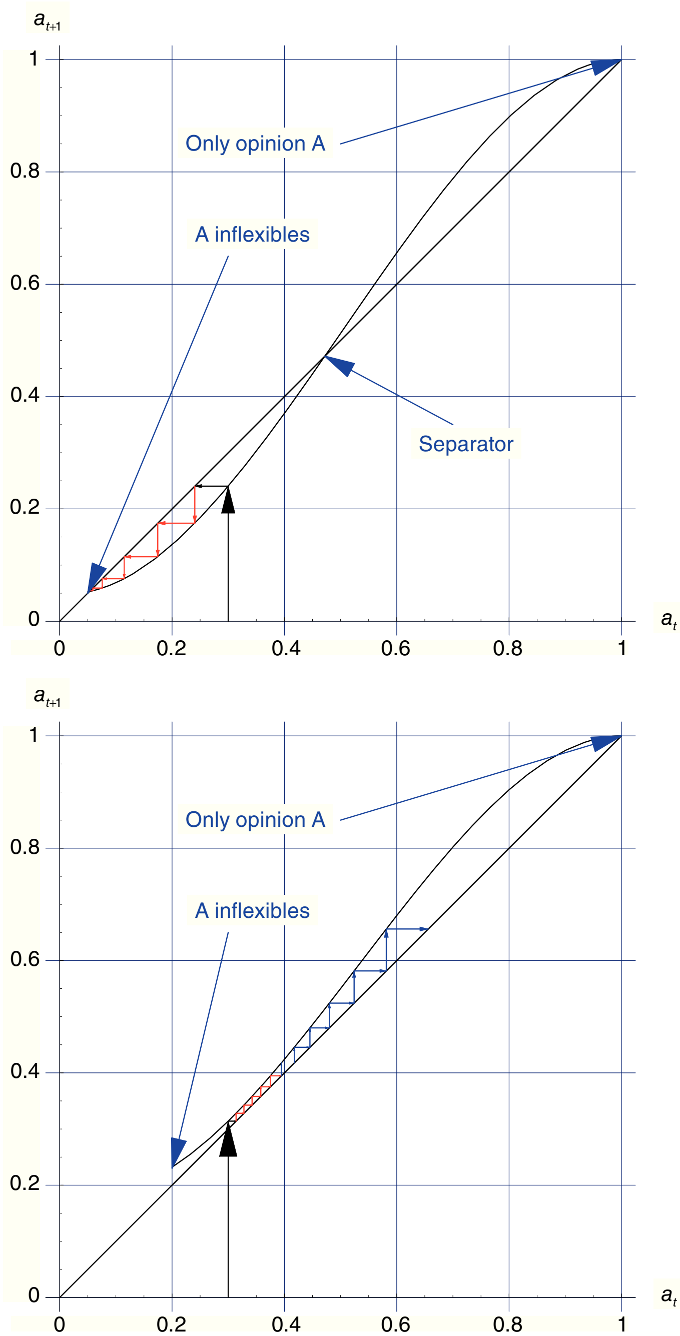

Figure 9. The evolution of a_{t+1} as a function of a_t from Eq. (9) for $q = 0.05 < q_c(3 - 2\sqrt{2}) \approx 0.172$ (left part) and $q = 0.20 > (3 - 2\sqrt{2}) \approx 0.172$ (right part). The evolution of $a_t = 0.30$ is shown in both cases. The first one leads to a very large B majority and a A minority while the second one produces a huge support in favor of opinion A, which eventually gains the full population support.

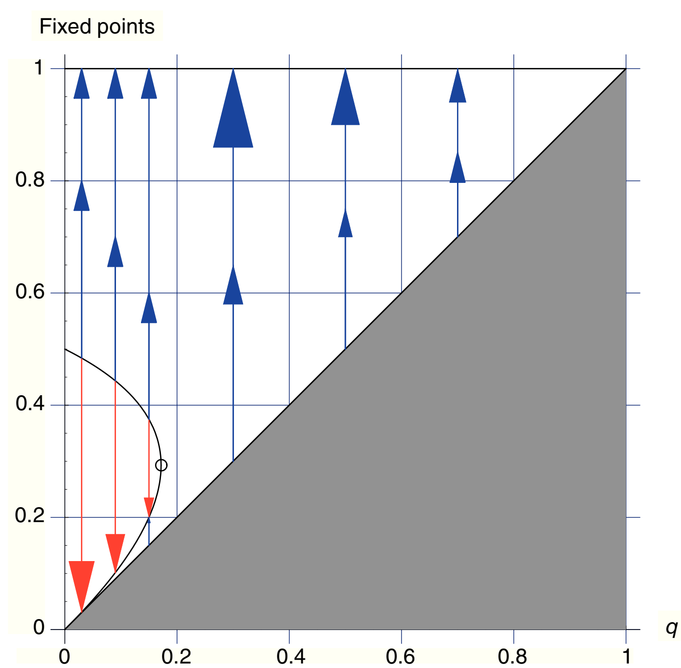

Figure 10. The three fixed points from Eq. (9) as a function of the inflexible density q. The attractor $a_A = 1$ is independand of q. At opposite both a_B^q (lower curve) and $a_{c,3}^q$ (middle curve) gets closer and closer with increasing q to merge and disappear at $q = (3 - 2\sqrt{2}) \approx 0.172$ with $a_B^q = a_{c,3}^q$ (little empty circle). The arrows show the direction of the opinion flow driven by the public debate. The grey area is the incompressible inflexible proportion of A opinion given by q.

spreading rumors. The model was used to explain the French hoax regarding 9/11 (Galam, 2003).

Moreover, in 2005, a highly improbable political vote outcome was predicted several months ahead of the actual vote (Galam, 2002, 2005c). The victory of the "no" to the French referendum on the European constitution was confirmed by the actual vote. It is worth stressing that early polls and all analyses had preicted a victory for the "yes" campaign. The heuristic power of sociophysics was clearly demonstrated as feasible. Nevertheless, one result is not sufficient to make definite conclusions, and more studies are needed.

In addition, our results about the highly non-linear effect of one-sided inflexibles may shed new light on some controversial issues for which no definitive scientific proof is available. An example is the global warming issue, for which there are many inflexibles to advocate the anthropic explanation. On the other hand, there are only a few people who refuse the human cause as not yet proven, but do not advocate a natural alternative with any certainty. In particular, the model shows how powerful the driving force of individual self-confidence is to get a very small minority to successfully reverse a huge opposite majority opinion.

In conclusion, we have presented a simple model that is able to reproduce some complexity of the social reality. The model suggests that the direction of the inherent polarization effect in the formation of a public opinion driven by a democratic debate is biased by the existence of common beliefs within a population. Homogeneous versus heterogeneous situations were shown to result in different qualitative outcomes.

Last, but not least, it is crucial to keep in mind that we are using models to mimic part of the reality. However, these models are only an approximation of that reality. Forgetting this important point may lead to some misunderstandings and misleading conclusions about what should be done to tackle the reality with a misuse of the approach. The limits of the approach must always be discussed before making any prediction. At this stage, the collaboration with researchers from social sciences could be valuable to make predictions on precise social events.

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