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A comparison of methodologies for fatigue analysis of shafts: DIN 743 vs. approaches based on Soderberg criterion

S.M.O. Tavares^{a,*}, P.M.S.T. de Castro^b

^aINEGI - Institute of Science and Innovation in Mechanical and Industrial Engineering, Porto, Portugal

^bFaculdade de Engenharia da Universidade do Porto, Porto, Portugal

Abstract

The design of rotating steel shafts is a classical mechanical engineering problem. Since the recognition of fatigue as a major source of failures in shafts, many different criteria for fatigue design of rotating steel shafts have been put forward. Two commonly used approaches are based on the Soderberg criterion and on the DIN 743 approach. However, in the vast and ever growing literature on fatigue design, comparisons of these two procedures, based on concrete examples, are not commonly available. Therefore, a clear need exists for a comparison of the two approaches. This article analyses these two approaches considering a simple and common case. This case is a transition in diameter of a steel shaft, assumed as the critical cross section where bending and torsion moments are applied. Contrary to expectation, substantial differences were found between the two approaches, including in the fatigue correction factors.

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1. Introduction

Power transmission elements are important machine components, subjected to continuous development aiming at better reliability and better efficiency. Within this group of elements, rotating shafts are commonly found. Their design constitutes a classical engineering problem and multiple approaches are available to determine the shaft characteristics. These design approaches, considering the shaft stress state and strength, are commonly divided in static and fatigue calculations [1].

Since the recognition of fatigue as a major source of failures in rotating shafts, many different criteria for fatigue design of rotating steel shafts have been put forward, from very approximate approaches up to

state-of-the-art multiaxial fatigue criteria. These advanced design criteria require a comprehensive computation of the stress field as a function of time during each loading cycle, as reviewed, e.g., by Anes *et al.* [2]. However, other approaches, directed to practical design (see, e.g., Bennebach *et al.* [3]), tend to be simpler to use and are particularly relevant in the early design phase where uncertainties may imply the need for numerous design iterations. In this context, two approaches originated in Germany should be mentioned: the DIN-743 [4–7] and the ‘FKM Guideline’ [8]. Examples of use of DIN 743 include Tavares *et al.* [9], Schmid *et al.* [10] and Opalić *et al.* [11], whereas McKelvey *et al.* [12], Behrens *et al.* [13] and Koechlin [14] report on the use of the ‘FKM Guideline’.

While simple approaches have inherent limitations of rigor, the more advanced approaches imply difficulties of application related to the specific material data, complex models used and to the heavy computational

* Corresponding author.

E-mail address: sergio.tavares@fe.up.pt (S.M.O. Tavares)

means involved. The effort and time required to perform the shaft design increases with the growth of complexity and accuracy of design, as pointed out by Ernzer and Birkhofer in [15], and schematically presented in Fig. 1.

Nevertheless, occasionally failures occur due to multiple reasons, possibly creating safety problems as in the case of failures of axles in metro or train vehicles, or in power transmission shafts for other applications.

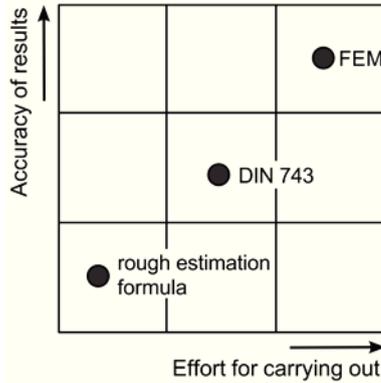


Fig. 1. Shaft design methodologies, effort vs. accuracy; adapted from [2].

Two common approaches that have been used to design rotating shafts based on standards are: (i) ANSI/ASME B106.1M – Design of Transmission Shafting [16], usually labelled as ASME method, and (ii) the German standard DIN 743, “Calculation of load capacity of shafts and axles”, [4-7].

2. ASME Method

ASME Design of Transmission Shafting is based upon a concept of static equivalent stress, as described in many US and English works on machine design, e.g., [17,18]. In this approach, the components of a multiaxial stress state creating fatigue, typically a cyclic normal stress combined with a shear stress that may or may not be cyclic, are firstly converted into equivalent static stresses using an approach such as Soderberg’s criterion. Once these values are obtained, a common static analysis follows, where the von Mises or the Tresca maximum shear stress criteria are applied using, for example, the Mohr circle. The multiaxial stress state is thus transformed into a uniaxial equivalent stress that is compared with the material yield strength divided by a safety factor, leading to the desired design: either the diameter value(s) given the bending and torsion moments, or a

quantification of applicable load(s), given specified value(s) for the diameter(s).

Considering the presentation proposed by Childs [17], and based on ASME approach, the shaft diameter can be determined as:

$$d = \left[\frac{32N}{\pi} \cdot \sqrt{\left(\frac{M_f}{\sigma_{f0}^c} \right)^2 + \frac{3}{4} \left(\frac{M_t}{\sigma_{yield}} \right)^2} \right]^{1/3} \quad (1)$$

where d is the shaft diameter (m), N the safety factor, M_f the bending moment (N.m), σ_{f0}^c the endurance limit (MPa), M_t the torque (N.m) and σ_{yield} the yield strength (MPa). This equation can be written as:

$$d = \left[\frac{32N}{\pi\sigma_{yield}} \cdot \sqrt{\left(\frac{\sigma_{yield} M_f}{\sigma_{f0}^c} \right)^2 + \frac{3}{4} M_t^2} \right]^{1/3} \quad (2)$$

or:

$$d^3 = \frac{32N}{\pi\sigma_{yield}} \cdot \sqrt{\left(\frac{\sigma_{yield} M_f}{\sigma_{f0}^c} \right)^2 + \frac{3}{4} M_t^2} \quad (3)$$

In order to understand the equivalent stress criteria adopted by ASME for shaft design, Eq. (3) can be written as:

$$\begin{aligned} \frac{\sigma_{yield}}{N} &= \sqrt{\left(\frac{\sigma_{yield}}{\sigma_{f0}^c} \cdot \sigma_a \right)^2 + 3 \cdot \tau^2} \\ &= \sqrt{\left(\sigma_{static}^{equivalent} \right)^2 + 3 \cdot \left(\tau_{static}^{equivalent} \right)^2} \end{aligned} \quad (4)$$

As a matter of detail, from Eq. (4) it is concluded that the ASME approach is based on von Mises criteria, and not in Tresca as frequently presented in Machine Elements courses namely at Faculty of Engineering of the University of Porto (FEUP), Portugal.

The endurance limit, σ_{f0}^c , is estimated as:

$$\sigma_{f0}^c = k_a k_b k_c k_d k_f k_g \sigma_{f0} \quad (5)$$

where k_a is the surface factor, k_b the size factor, k_c the reliability factor, k_d the temperature factor, k_e the duty cycle factor, k_f the fatigue stress concentration factor, k_g the miscellaneous effects factor and σ_{f0} the endurance limit of test specimen (in MPa).

For each case, these different factors can be obtained from tables and expressions in the ASME standard or machine design literature [17]. Concerning the fatigue

stress concentration factor, k_f , in the notation of Eq. (5), this is determined by:

$$k_f = \frac{1}{K_f} \quad (6)$$

where K_f is given by:

$$K_f = 1 + q(K_t - 1) \quad (7)$$

and q is the notch sensitivity and K_t the geometric stress concentration factor.

For steel shaft design, the endurance limit can be approximated as:

$$\sigma_{f0} = 0.504 \cdot \sigma_{UTS} \text{ for } \sigma_{UTS} \leq 1400 \text{ MPa} \quad (8)$$

and

$$\sigma_{f0} = 700 \text{ MPa for } \sigma_{UTS} > 1400 \text{ MPa} \quad (9)$$

3. DIN 743

Shaft design according to the German standard DIN 743 [4-7] implies an analysis for safety against fatigue, and a separate analysis concerning safety against plastic deformation. The first, analysis against fatigue, is based upon the amplitude of the normal stress and the amplitude of the shear stress; normal stress being further separated into its components resulting from axial load and from bending. For all these components, the corresponding strength, taking into account the relevant R values (R being the ratio minimum/maximum value of the stress under consideration), is obtained from appropriate Smith diagrams. In the second case - analysis concerning safety against plastic deformation -, the maximum values of the several components of stress are considered, and the strength values come from static testing, where in particular the difference between yield and rupture strengths for tensile stress, resulting from axial and from bending loads, is taken explicitly into account.

The definition of a safety factor against fatigue assesses the stress amplitudes in tension/compression, bending and torsion applied and the respective permissible stress amplitudes for that shaft (equivalent to the shaft strength). This safety factor is given by:

$$S = \frac{1}{\sqrt{\left(\frac{\sigma_{zda}}{\sigma_{zADK}} + \frac{\sigma_{ba}}{\sigma_{bADK}}\right)^2 + \left(\frac{\tau_{ta}}{\tau_{tADK}}\right)^2}} \quad (10)$$

where σ_{zda} , σ_{ba} and τ_{ta} are the stress amplitudes due to tension/compression, bending and torsion, respectively

and σ_{zADK} , σ_{bADK} and τ_{tADK} the permissible stress amplitudes, taking into account the material fatigue strength for tension/compression, bending and torsion, respectively.

To determine the permissible stress amplitudes, this standard considers two different cases of shaft design:

- Case 1: the mean stress (σ_{mv}) is constant;
- Case 2: the load ratio ($R = \sigma_{min}/\sigma_{max}$) is constant.

In case 1, the permissible fatigue stress are:

$$\begin{aligned} \sigma_{zADK} &= \sigma_{zDWK} - \psi_{z\sigma K} \cdot \sigma_{mv} \\ \sigma_{bADK} &= \sigma_{bWK} - \psi_{b\sigma K} \cdot \sigma_{mv} \\ \tau_{tADK} &= \tau_{tWK} - \psi_{t\tau K} \cdot \tau_{mv} \end{aligned} \quad (11)$$

where the terms with index WK are the fatigue strength of the notched shaft, determined by:

$$\begin{aligned} \sigma_{zDWK} &= \frac{\sigma_{zW}(d_B) \cdot K_1(d_{eff})}{K_\sigma} \\ \sigma_{bWK} &= \frac{\sigma_{bW}(d_B) \cdot K_1(d_{eff})}{K_\sigma} \\ \tau_{tWK} &= \frac{\tau_{tW}(d_B) \cdot K_1(d_{eff})}{K_\tau} \end{aligned} \quad (12)$$

where $K_1(d_{eff})$ is the technological size factor for the effective diameter, K_σ and K_τ the fatigue factors for tension/compression/bending and torsion. The fatigue strengths (terms with index W) are determined as:

$$\begin{aligned} \sigma_{zW}(d) &= 0.4 \cdot \sigma_B(d) \\ \sigma_{bW}(d) &= 0.5 \cdot \sigma_B(d) \\ \tau_{tW}(d) &= 0.3 \cdot \sigma_B(d) \end{aligned} \quad (13)$$

where $\sigma_B(d)$ is the tensile strength of the shaft material for the diameter used, which can be determined as:

$$\sigma_B(d) = K_1(d) \cdot \sigma_B(d_B) \quad (14)$$

where $K_1(d)$ is the technological size factor and $\sigma_B(d_B)$ the tensile strength for a reference diameter. The ψ factors, for the case 1, are determined as:

$$\begin{aligned} \psi_{z\sigma K} &= \frac{\sigma_{zDWK}}{2 \cdot K_1(d_{eff}) \cdot \sigma_b(d_b) - \sigma_{zDWK}} \\ \psi_{b\sigma K} &= \frac{\sigma_{bWK}}{2 \cdot K_1(d_{eff}) \cdot \sigma_b(d_b) - \sigma_{bWK}} \\ \psi_{t\tau K} &= \frac{\tau_{tWK}}{2 \cdot K_1(d_{eff}) \cdot \sigma_b(d_b) - \tau_{tWK}} \end{aligned} \quad (15)$$

The K_σ and K_τ factors can be determined as:

$$\begin{aligned} K_\sigma &= \left(\frac{\beta_\sigma}{K_2(d)} + \frac{1}{K_{F\sigma}} - 1 \right) \cdot \frac{1}{K_V} \\ K_\tau &= \left(\frac{\beta_\tau}{K_2(d)} + \frac{1}{K_{F\tau}} - 1 \right) \cdot \frac{1}{K_V} \end{aligned} \quad (16)$$

where $K_2(d)$ is the geometrical size factor, β_σ and β_τ the notch factors, $K_{F\sigma}$, $K_{F\tau}$ the surface roughness factors and K_V the surface hardening factor.

4. Case Study

For assessment of both shaft design procedures, a simple example is fully worked out using both methods. The example is proposed by Childs [17] and it consists of a transition in diameter of a steel rotating shaft, where in the assumed critical cross section a bending and a torsion moment are applied, creating a cyclic normal stress with $R = -1$ and a constant shear stress (Fig. 2). It is also considered that the shaft is in steel, with strength values $\sigma_{yield} = 770$ MPa and $\sigma_{UTS} = 1000$ MPa. The following moments act in this cross section, assumed to be the critical one: $M_{bending} = 158.5$ N.m and $M_{torsion} = 84.9$ N.m.

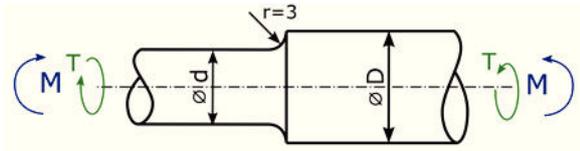


Fig. 2. Schematic representation of the shaft used in this case study.

4.1. ASME approach

The endurance limit of the shaft material is $\sigma_{f0} = 0.504 \cdot \sigma_{UTS} = 504$ MPa. Considering hot rolled condition, the surface factor is determined as

$$k_a = a \cdot \sigma_{rupture}^b \quad (17)$$

with $a = 57.5$ and $b = -0.718$, i.e. $k_a = 0.403$.

The size factor, assuming $d = 30$ mm:

$$k_b = (d/7.62)^{-0.1133} = 0.856 \quad (18)$$

Reliability factor (shaft nominal reliability assumed as 90%), $k_c = 0.897$. Temperature effect was not considered, therefore $k_d = 1$. Duty cycle factor is assumed $k_e = 1$.

The stress concentration factor, determined by Eqs. (6) and (7) and considering $r/d = 3/30 = 0.1$,

$D/d = 36/30 = 1.2$, the stress concentration factor is $K_t = 1.65$, the notch sensitivity factor is $q = 0.9$ and, therefore, $k_f = 0.631$. Since other effects are not considered, the miscellaneous effects factor is taken as $k_g = 1$.

The endurance limit, considering Eq. (5) for this case, is:

$$\sigma_{f0}^c = 98.5 \text{ MPa} \quad (19)$$

Considering a safety factor of 2 in Eq. (1), the diameter $d = 32$ mm is obtained.

4.2. DIN 743 approach

Unlike the ASME procedure, the DIN 743 procedure does not calculate specifically the minimal possible diameter of the shaft; instead, the DIN 743 standard procedure proves, for given conditions of geometry, material, etc., if a given safety factor is achieved. In order to compare both design methods, a calculation of safety factor according to DIN 743 will be carried out using $d = 32$ mm.

The input of the analysis is $d = 32$ mm (resulting from the ASME calculation), $D = 38$ mm, $r = 3$ mm, $t = 3$ mm, bending moment $M_b = 158.8$ N m, torsion moment $M_t = 84.9$ N m, implying:

$$\sigma_b = \sigma_{ba} = \frac{M_b}{W_{ax}} = 43.36 \text{ MPa}$$

and

$$\tau_t = \frac{M_t}{W_p} = 13.2 \text{ MPa}$$

Concerning the material, Childs [17] mentions the 817M40 hot-rolled alloy steel; this is equivalent to German 34CrNiMo6 steel, with $\sigma_{UTS} = \sigma_B = 1000$ MPa and $\sigma_{yield} = \sigma_S = 770$ MPa, therefore the fatigue strength for $R = -1$, according to Eq. (13) in bending $\sigma_{bW} = 500$ MPa, in tension/compression $\sigma_{zdW} = 400$ MPa and $\tau_{bW} = 300$ MPa in torsion.

The assumed surface roughness is $R_z = 5$ μm .

The bending factor is given by:

$$\alpha_\sigma = 1 + \frac{1}{\sqrt{0.62 \cdot \frac{r}{t} + 11.6 \cdot \frac{r}{d} \cdot \left(1 + 2 \cdot \frac{r}{d}\right)^2 + 0.2 \left(\frac{r}{t}\right)^3 \cdot \frac{d}{D}}} \quad (20)$$

For this case, $\alpha_\sigma = 1.656$. The stress gradient is:

$$G' = 2.3 \cdot (1 + \varphi) / r = 0.895 \text{ mm}^{-1}$$

where $\varphi = 1 / (4 \cdot \sqrt{t/r} + 2) = 0.167$.

The technological size factor, for $d = 32$ mm, is $K_1(d_{eff}) = 0.9$.

The notch sensitivity factor is given by:

$$n = 1 + \sqrt{G' \cdot \text{mm}} \cdot 10^{-\left(0.33 + \frac{\sigma_S(d)}{712 \text{ MPa}}\right)} \quad (21)$$

which, for the present case, with $\sigma_S(d) = 0.9.770 = 693$ MPa, results in $n = 1.047$. The notch sensitivity factor is:

$$\beta_\sigma = \frac{\alpha_\sigma}{n} = 1.581 \quad (22)$$

The geometrical size effect $K_2(d)$, is equal to:

$$K_2(d) = 1 - 0.2 \cdot \frac{\log(d/7.5 \text{ mm})}{\log 20} = 0.903 \quad (23)$$

The surface roughness factor is:

$$K_{F\sigma} = 1 - 0.22 \cdot \log\left(\frac{R_z}{\mu\text{m}}\right) \cdot \left[\log\left(\frac{\sigma_B(d)}{20 \text{ Nmm}^{-2}}\right) - 1\right] \quad (24)$$

Since $\sigma_B(d) = 900$ MPa (Eq. (14)), $K_{F\sigma} = 0.9$.

The surface hardening factor of $K_v = 1$ is assumed. Therefore, the total fatigue factor, Eq. (16), is $K_\sigma = 1.863$. The torsion factor, α_τ , is given by:

$$\alpha_\tau = 1 + \frac{1}{\sqrt{3.4 \cdot \frac{r}{t} + 38 \cdot \frac{r}{d} \cdot \left(1 + 2 \cdot \frac{r}{d}\right)^2 + \left(\frac{r}{t}\right)^2 \cdot \frac{d}{D}}} \quad (25)$$

for the present case equal to $\alpha_\tau = 1.329$. The stress gradient for torsion is given by $G' = 1.15/r = 0.383 \text{ mm}^{-1}$. The technological size factor is also equal to $K_1(d_{eff}) = 0.9$. The notch sensitivity factor is $n = 1.03078$ (considering $\sigma_S(d) = 693$ MPa) and the notch effect coefficient is $\beta_\tau = 1.329/1.031 = 1.289$. The geometrical size effect is also the same, Eq. (23), equal to $K_2(d) = 0.903$.

The surface roughness factor for torsion is given by:

$$K_{F\tau} = 0.575 \cdot K_{F\sigma} + 0.425 = 0.942 \quad (26)$$

The surface hardening factor of $K_v = 1$ is assumed.

The total fatigue torsion factor is $K_\tau = 1.488$.

The mean stress is given by:

$$\sigma_{mv} = \sqrt{(\sigma_{zdm} + \sigma_{bm})^2 + 3\tau_{tm}^2} \quad (27)$$

Since $\sigma_{zdm} = 0$, $\sigma_{bm} = 43.37$ MPa and $\tau_{mv} = \sigma_{mv}/\sqrt{3} = 31.41$ MPa the mean stress is $\sigma_{mv} = 54.39$ MPa.

From Eq. (12), the fatigue strength for this shaft is:

$$\sigma_{zdwk} = 241.6 \text{ MPa}; \quad \tau_{twk} = 181.4 \text{ MPa} \quad (28)$$

and the factors ψ , from Eq. (15), are:

$$\psi_{b\sigma K} = 0.155; \quad \psi_{\tau K} = 0.112 \quad (29)$$

then, the permissible fatigue stress, Eq. (11):

$$\sigma_{bADK} = 233.95 \text{ MPa}; \quad \tau_{tADK} = 179.9 \text{ MPa} \quad (30)$$

Therefore, the safety factor, according to Eq. (10), is:

$$S = 4.74 \quad (31)$$

If this safety factor of 4.74 is used to estimate the shaft diameter according to ASME standard, Eq. (1), a diameter of 43 mm would be obtained.

5. Conclusions

An assessment of the best-known shaft design standards ASME B106.1M and DIN 743 is presented. Both approaches have considerable differences, DIN 743 being more comprehensive since it divides the fatigue effect for tension/compression and bending to the torsion effect. However, the biggest differences were found in the correction factors as the surface factor presented in Fig. 3.

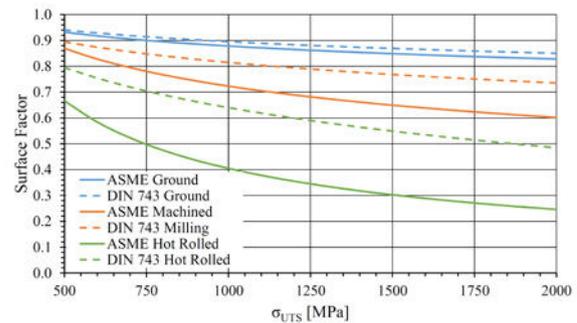


Fig. 3. Comparison of the surface factor based on ASME and on DIN 743 approaches.

Since the case study is based on Childs example presented in [17], where the shaft is in hot rolled condition, considerable difference is found in the surface factor adopted.

The suggested safety factors for use with the two approaches are substantially different: for ASME safety factors 1.25 to 4.0 are suggested, whereas DIN suggests only a minimum safety factor of 1.2.

The need for improved understanding of the sources of differences, and for establishing the limitations of each approach, was a result of the work presented here. It is suggested that a systematic exercise, based on well-

documented experimental situations, should be carried out comparing the different approaches, including the more fundamental high cycle multiaxial fatigue analysis models, so that a deeper knowledge of the advantages and drawbacks of each approach is made clearer.

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